Solutions to Homework 1

AEC 504 - Summer 2007

Fundamentals of Economics

1 Hard Charity

Fairy Godmother has recently retired and moved from Far Far Away, CA to Rochester, NY. Fairy Godmother receives her retirement benefit by-weekly. Every second Friday evening she cashes in her paycheck of $1,500 and drives to Pittsford Plaza, where she buys some consumer goods and donates the rest to Salvation Army. Her utility function is $U = G^{2/3}D^{1/6}$, where G is the dollar value of consumer goods purchased and D is the number of $10 bills donated.

i. Compute the optimal allocation of Fairy Godmother's money between purchases and donations

Standard solution:

As you can check by taking partial derivatives, Fairy Godmother's marginal utility of a unit of consumer goods (a $10 bill donated) is equal to $\frac{2}{3}G^{-1/3}D^{1/6}$ ($\frac{1}{6}G^{2/3}D^{-5/6}$)

Optimal allocation is obtained by setting the MRS equal to the ratio of prices:

$$\frac{MU_G}{MU_D} = \frac{P_G}{P_D} \Rightarrow \frac{2/3 \cdot G^{-1/3}D^{1/6}}{1/6 \cdot G^{2/3}D^{-5/6}} = \frac{1}{10} \Rightarrow \frac{4D}{G} = \frac{1}{10} \Rightarrow G = 40D \tag{1}$$

Because G is money and D is ten-dollar bills, the budget constraint is:

$$P_G \cdot G + P_D \cdot D = I \Rightarrow G + 10D = 1500 \tag{2}$$

Now substitute the optimal allocation condition (1) into the budget constraint (2) and get the answer:

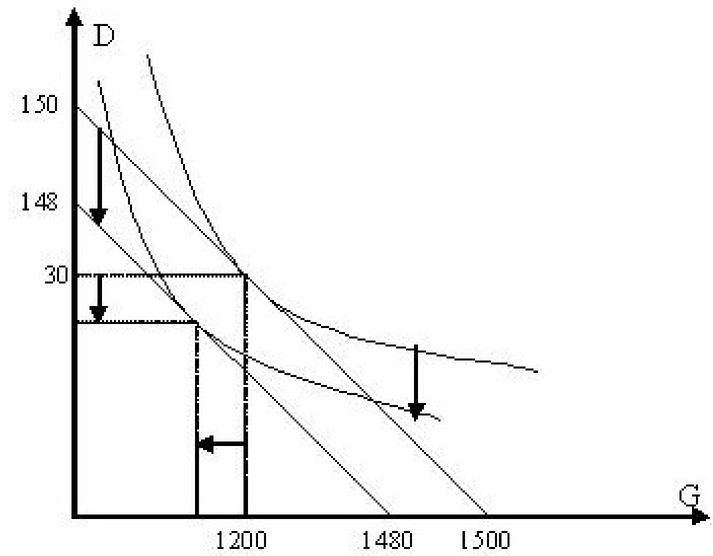
$$40D + 10D = 1500 \Rightarrow D = 1500/50 = 30 \Rightarrow G = 40 \cdot 30 = 1200 \tag{3}$$

Shortcut for Cobb-Douglas utility:

$$U = G^{\alpha} D^{\beta} \Rightarrow G = \frac{\alpha \cdot I}{P_G \cdot (\alpha + \beta)}; \quad D = \frac{\beta \cdot I}{P_D \cdot (\alpha + \beta)} \Rightarrow$$
$$\Rightarrow G = \frac{\$1500 \cdot 2/3}{\$1 \cdot (2/3 + 1/6)} = 1200; \quad D = \frac{\$1500 \cdot 1/6}{\$10 \cdot (2/3 + 1/6)} = 30 \qquad (4)$$

Answer: Fairy Godmother spends $1200 on consumer goods and donates $300

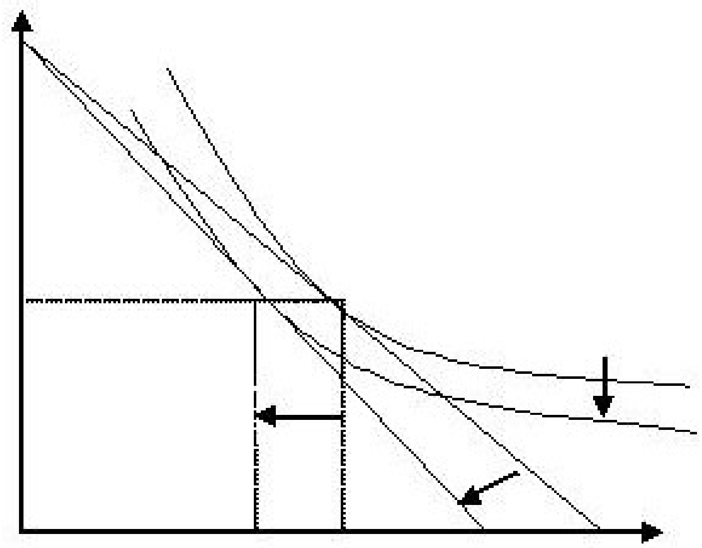
ii. Suppose now that the charity gatherers have left Pittsford Plaza, and Fairy Godmother has to drive to downtown Saturday morning to make her donation. The trip costs her $20 (including the price of Medieval combo, which brings her no utility whatsoever). Show graphically what happens to the optimal allocation



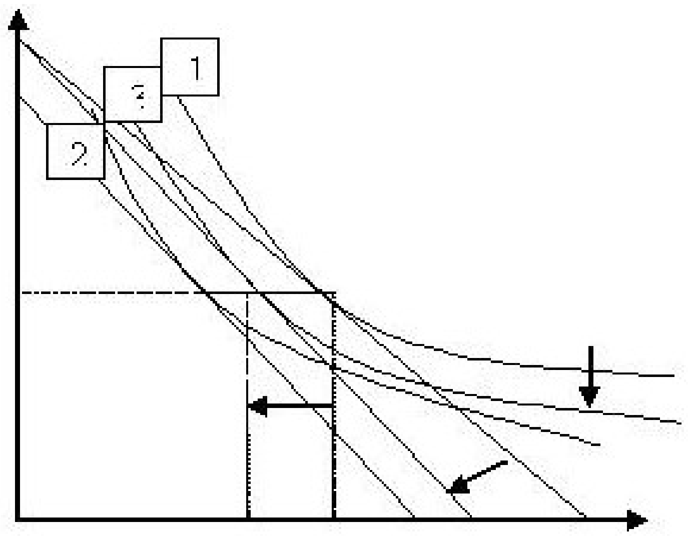
Fairy Godmother will reduce both the amount donated and the amount of goods purchased

iii. Suppose now that Salvation Army accepts credit cards, but charges 5% fee for a donation made via credit card. Show graphically what happens to the optimal allocation. Draw a graph showing that it is better for Fairy Godmother to pay with credit card than to drive downtown and pay no percentage fee (you do not have to prove that she is really better off paying with credit card, but you graph should show she is)

Fairy Godmother donates less, but it is not clear without additional analysis if she consumes more or less – substitution effect pushes her consumption upwards, income effect pulls it downwards



The point here is to draw indifference curve 2 lower than 3 and to make the optimal bundle under the budget constraint from (iii) unattainable under the budget constraint from (ii).



iv. Is there any fee schedule (fixed, as in (ii), percentage, as in (iii), or a combination of both), which would make Fairy Godmother donate nothing?

The only way to make Fairy Godmother donate nothing is to take away all her income if she decides to donate anything. There are two ways to explain it. First, one can notice that she will have U=0 if she donates nothing, irrespectively of the amount of goods consumed. Thus, she will always want to donate something if she has anything to donate. Second, one can differentiate her utility function wrt D and see that the marginal utility of donation equals infinity if she does not donate, that is, she would donate a cent at any cost of donation.

2 Much Ado about a Budget Line

A schoolgirl spends her pocket money of \$20 per week on movie tickets and sundaes. A movie ticket costs \$5, and a sundae costs \$2. Show graphically the change in her budget constraint for the following events (consider one event at a time):

i. The price of a sundae increases to \$2.50.

ii. The schoolgirl finds a \$5 bill on the pavement.

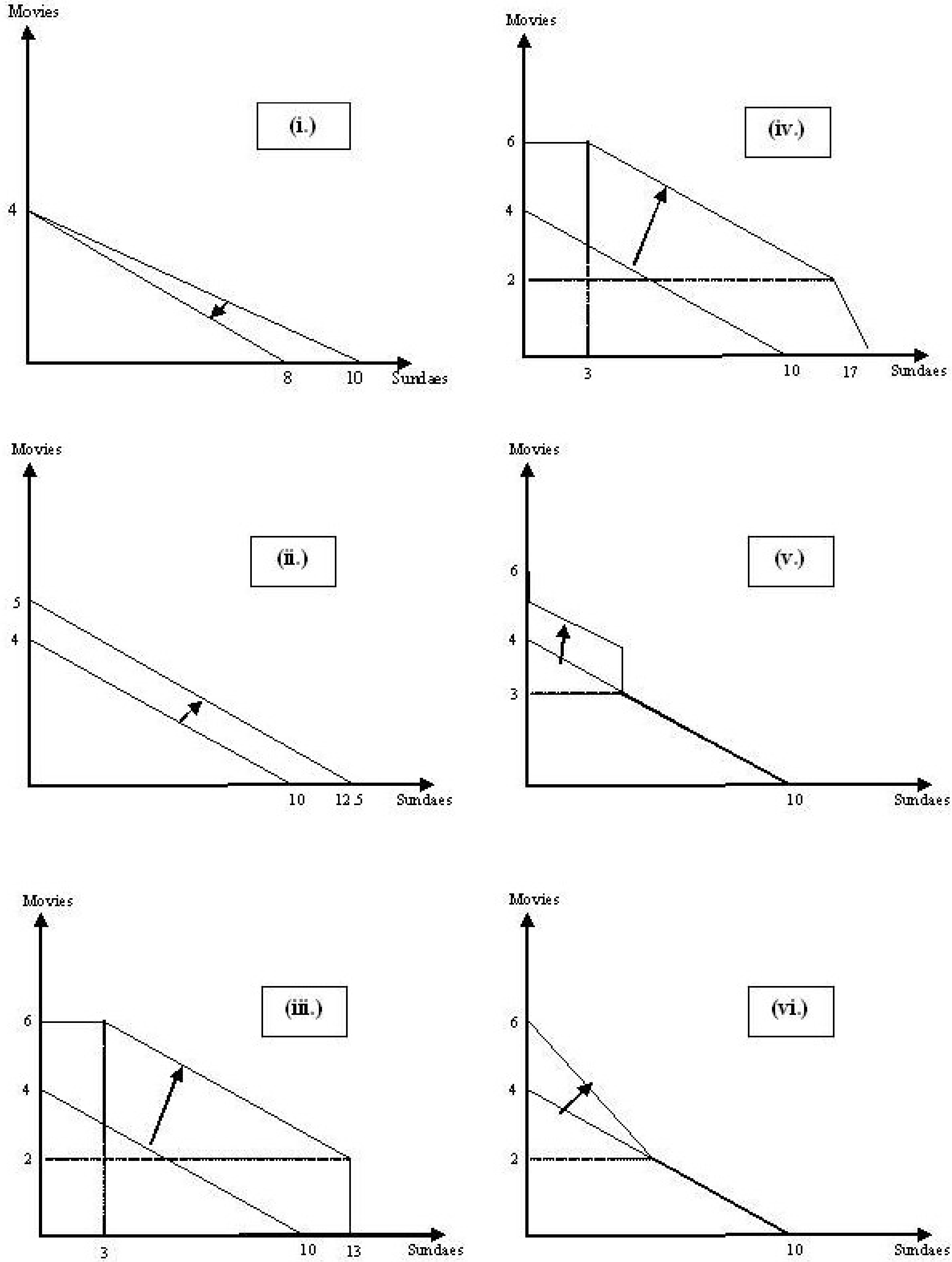
iii. In addition to the pocket money, Dad buys her two movie tickets and three sundaes a week. The girl cannot sell what Dad gives her to somebody else.

iv. Being busy, Dad does not go to see the movie with her, and she figures out she can sell the tickets Dad buys her to her classmate for \$4. The sundaes Dad buys her still have no resale value.

v. The movie theater introduces a discount program: for everyone buying more than two movie tickets per week each movie ticket starting with the third one gives the right to see another movie for free.

vi. The movie theater changes the discount program to give everyone buying more than two tickets per week 50% discount on the tickets purchased afterwards.

P.S. Fractional sundaes and movie tickets are OK.



<u>Some comments</u>: In (iii), note the horizontal and the vertical part. They mean, respectively, that the girl does not gain anything by giving up the first three sundaes and the first two movie tickets Dad buys her, because the gifts are non-tradable. In (iv), note the kink in the bottom - it appears because she has to sell the first two tickets (Dad's gift) at a discount, so the budget line is steeper (on this portion she has to give up more movie tickets to buy the same number of sundaes). In (v), the vertical parts again mean that giving up the free tickets the movie theater gives her (the 4th one and the 6th one) does not buy her more sundaes.

3 Catch-22

Captain Yossarian makes \$6,000 per year and spends it on buying wine and making trips to Rome. A case of wine, W, costs \$100 and a trip to Rome, R, costs \$300. The utility function of Yossarian is $U = \sqrt{W} + \sqrt{R}$. (Fractional trips and cases are OK).

i. How many cases of wine does Yossarian buy per year? How many trips to Rome he makes?

$$\begin{cases} \frac{MU_W}{MU_R} = \frac{p_W}{p_R} \\ 100W + 300R = 6000 \end{cases} \Rightarrow \begin{cases} \sqrt{\frac{R}{W}} = \frac{1}{3} \\ 100 \cdot 9R + 300R = 6000 \end{cases} \Rightarrow \begin{cases} W = 45 \\ R = 5 \end{cases}$$

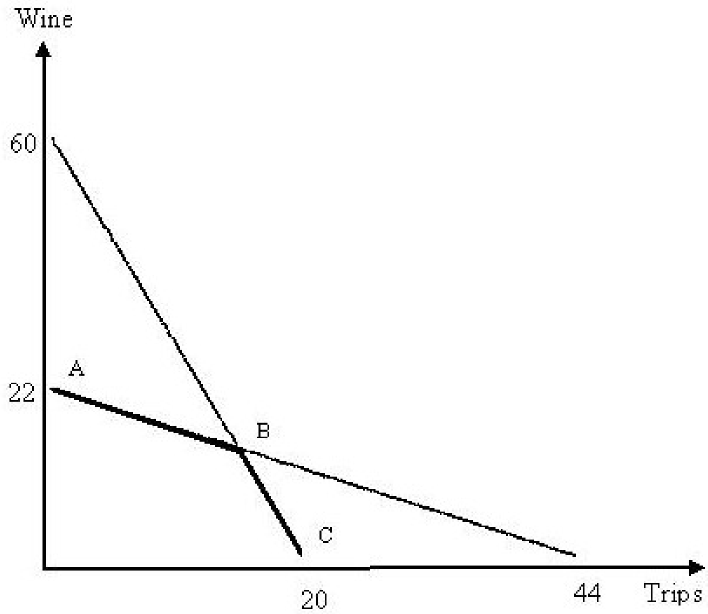
Yossarian buys 45 cases of wine and makes 5 trips to Rome per year. (No wonder the book starts with Yossarian having liver pains.)

ii. Colonel Cathcart notices that officers cannot fly combat missions while they are drinking wine or are making trips to Rome. Therefore, he endows each officer with exactly 22 coupons for a year. A coupon buys a case of wine or two trips to Rome, besides the money to be paid. The coupons are not tradable. Draw Yossarian's budget constraint. Compute his optimal consumption bundle after the coupons are introduced.

The budget constraint is now modified to include only the consumption bundles Yossarian has enough money and coupons to buy. The algebraic

version and the graph (the budget constraint is the kinked line ABC) are below:

$$\begin{cases} 100W + 300R \leq 6000 \\ W + 0.5R \leq 22 \end{cases}$$



The algorithm to solve the problem is the following. First check if Yossarian has enough coupons to buy his optimal bundle from (i). If yes, bingo! - by revealed preferences, it is the solution to (ii) as well. (In the picture, AB is within his old budget constraint, so if his optimal consumption bundle is on BC, then he will not want to be anywhere on AB by definition of constrained optimum).

In our case, he does not have enough coupons - 45 cases of wine alone call for 45 coupons, and he only has 22. So, we make the second check. We maximize his utility subject to the coupon constraint alone and see if he has enough money to buy this bundle (i.e. if this bundle is on AB). If he does, then by revealed preferences he will not want anything

on BC, and this is the solution.

$$\begin{cases} \dfrac{MU_W}{MU_R} = \dfrac{p_W}{p_R} \\ W + 0.5R = 22 \end{cases} \Rightarrow \begin{cases} \sqrt{\dfrac{R}{W}} = 2 \\ W + 0.5 \cdot 4W = 22 \end{cases} \Rightarrow \begin{cases} W = \dfrac{22}{3} \\ R = \dfrac{88}{3} \end{cases}$$

If he does not have enough money to buy the bundle (and he does not - 29 trips to Rome alone require $6,000), then the solution is the corner (point B). The coordinates of point B are found by solving the system including both constraints as equalities (in our case, subtract the coupon constraint from the money constraint divided by 100).

$$\begin{cases} W + 3R = 60 \\ W + 0.5R = 22 \end{cases} \Rightarrow \begin{cases} W = 14.4 \\ R = 15.2 \end{cases}$$

Answer: After colonel Cathcart introduces the coupons, Yossarian cuts the wine consumption to 14.4 cases and increases the number of trips to Rome to 15.2.

P.S. You can get the same answer by formally maximizing the utility function subject to the two constraints and applying the Kuhn-Tucker conditions.

iii. Milo Minderbinder somehow gets an unlimited number of coupons and begins to trade in them at $50 per coupon. Redraw Yossarian's budget constraint. Will Yossarian buy or sell coupons? What is his optimal consumption bundle now?

If coupons are freely traded, they are just money. Yossarian income increases to $7,100 - he can now sell his endowment of 22 coupons at $50 each and make $1,100. The price of wine is $150 - $100 plus $50 to buy the coupon, and the price of the trip to Rome is $325. Hence, the new budget constraint is $150W + 325R = 7100$***, and the solution is analogous to (i):***

$$\begin{cases} \dfrac{MU_W}{MU_R} = \dfrac{p_W}{p_R} \\ 150W + 325R = 7100 \end{cases} \Rightarrow \begin{cases} \sqrt{\dfrac{R}{W}} = \dfrac{150}{325} \\ 150W + 325 \cdot \dfrac{36}{169}W = 7100 \end{cases} \Rightarrow \begin{cases} W \approx 32.4 \\ R \approx 6.9 \end{cases}$$

Yossarian will have to buy coupons from Milo, because his wine consumption alone calls for 32.4 coupons.

iv. Use the revealed preference principle to rank Yossarian's wealth in (i), (ii), and (iii).

Point B is on the money constraint and on the coupon constraint, because it is their intersection. So, the optimal bundle from (ii) (point B) is clearly attainable in (i), but it is not chosen. By revealed preference Yossarian is better off in (i) than in (ii).

Point B is also on the budget constraint in (iii) - Yossarian can always refuse to deal with Milo and stick to point B. But he chooses to buy coupons from Milo, which implies that he is better off doing that, so his wealth in (iii) is bigger than in (ii).

We have to substitute (iii) outcome in the money constraint to see that it is attainable in (i), but not chosen, so Yossarian is better off in (i) than in (iii). The final ranking is therefore $(i) \succ (iii) \succ (ii)$.

v. Disaggregate Yossarian's change in the wine consumption and the trips to Rome from (i) to (iii) into the income effect and the substitution effect. Support the numerical answers with economic intuition.

Let's figure out the income effect first. To do that, we need to bring Yossarian to the old utility level of $\sqrt{45} + \sqrt{5} = 4\sqrt{5}$ ***at the new prices of \$150 per case of wine and \$325 per trip to Rome, i.e. minimize the expenditure of*** $150R + 325W$ ***needed to achieve the utility level of*** $4\sqrt{5}$. ***The change in*** W ***and*** R ***between (iii) and the expenditure minimization solution is the income effect. The rest is the substitution effect.***

$$\begin{cases} \dfrac{MU_W}{MU_R} = \dfrac{p_W}{p_R} \\ \sqrt{W} + \sqrt{R} = 4\sqrt{5} \end{cases} \Rightarrow \begin{cases} \sqrt{\dfrac{R}{W}} = \dfrac{150}{325} \\ \dfrac{13}{6}\sqrt{R} + \sqrt{R} = 4\sqrt{5} \end{cases} \Rightarrow \begin{cases} W \approx 37.45 \\ R \approx 7.98 \end{cases}$$

So, the income effect is $6.9 - 7.98 = -1.08$ ***for trips to Rome and*** $32.4 - 37.45 = -5.05$ ***for wine.***

<u>Intuition</u>: We found in (iv) that Yossarian is worse off in (iii) compared to (i), so the income necessary to reach the utility level in (iii) is smaller than the income necessary to reach the utility level in (i). That is,

the income effect for both goods has to be negative assuming they are normal.

NB: Note that Yossarian can buy more trips to Rome with his income in (iii) than in (i), (21.84 vs 20), so he seems "richer" in terms of the trips, yet the income effect for the trips is negative, not positive. Conclusion: do not use this type of reasoning to figure out the sign of the income effect.

The substitution effect is $37.45 - 45 = -7.55$ ***for wine and*** $7.98 - 5 = 2.98$ ***for trips to Rome.***

<u>Intuition</u>: In (i), a trip to Rome costs 3 cases of wine, and in (iii) it costs only $325/125 = 2.8$ ***cases of wine. Therefore, the substitution effect is positive for trips to Rome (they became cheaper in terms of wine) and negative for wine.***