

Relative Gain Maximization in Sequential 3-Person Characteristic Function Games

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The present study tested competitively three descriptive models of coalition formation and payoff disbursement in sequential 3-person games in which each player seeks to maximize the rank of his or her total score in a sequence of interdependent characteristic function games with sidepayments. Two of the models were originally proposed and tested by J. D. Laing and R. J. Morrison. A third mixed-signal model is proposed, postulating that the starting rank position and the values of the characteristic function, which operate as two independent signals, are combined to determine both coalition frequencies and payoff division. To test these models, 25 subjects played 46 different sequences for a total of 236 games in a new experimental paradigm, which generalizes previous research by assigning different values to the three 2-person coalitions, introducing dependency between successive characteristic functions, and eliminating face-to-face bargaining. The results support the mixed-signal model over its competitors. © 1985 Academic Press, Inc.

In experimental studies of mixed-motive conflicts, subjects are typically instructed to maximize their own individual gain and are subsequently paid proportionally to their total score. There is strong evidence that subjects do not always adhere to these unambiguous instructions. The results of experiments conducted by McClintock and McNeel (1966, 1967) suggest that in mixed-motive contexts represented by 2-person nonnegotiable nonzero-sum games, players do not only try to maximize their payoffs, but are, in fact, more concerned with their scores relative to the other player than with the magnitude of their own individual scores (Messick & McClintock, 1968). In such situations, therefore, it may be adequate to describe

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person i 's payoff, T_i , as $T_i = f(P_i, D_i)$, where P_i denotes i 's absolute gain, D_i denotes i 's relative gain ($D_i = P_i - P_j$), and the function f is monotonically increasing in each of its two arguments. In a series of experiments, Messick and Thorngate (1967) demonstrated that subjects do, in fact, tend to maximize relative gain when knowledge of the other person's payoffs is available.

Although the experimental evidence referred to above has been gathered in 2-person interactions, relative gain maximization may also be a major source of motivation in groups with three or more persons. In social situations such as class examinations, various sport competitions, and many group experiments, some individuals strive to be good at that which they do, others attempt to be better than their peers, and yet others try to be best (Shubik, 1971). Indeed, many social situations are designed to render relative gain maximization or, equivalently, rank position maximization of paramount importance to the participants. For example, at military academies such as West Point and at many schools in France, performance traditionally has been publicly recorded in terms of final class standing at graduation. The presidential primary system in the United States is another instance in which maximization of final rank position is the ultimate goal (Chertkoff, Skov, & Catt, 1980). And in the Olympic games, the gold, silver, and bronze medals are awarded to individual athletes or teams (e.g., basketball) ranked first, second, and third, respectively.

Realizing that status or social position is often more important in a society than wealth or other physical goods, Shubik (1971) introduced a class of games of status, which are characteristic function (CF) games with side payments (Kahan & Rapoport, 1984; Luce & Raiffa, 1957; Rapoport, 1970; Shubik, 1982) in which the true payoff of the game is not the score an individual player obtains but, rather, the player's status, which is determined by the rank order of the amount he or she obtains. Whereas Shubik focused on single-trial games of status, Laing and Morrison (1973, 1974a, 1974b) introduced a special class of multi-trial or sequential 3-person games of status in which the three players participate in a sequence of interdependent CF games (trials) and get paid according to their final rank. Laing and Morrison extended several game-theoretic solution concepts to their sequential 3-person game of status and, as an alternative, proposed two heuristic models to predict coalition frequencies and payoff allocation among status striving players. The two models, called *myopic* and *hyperopic*, are based on the assumption that players are unable to represent the overall structure of the sequential 3-person game of status in all its complexity. Rather, in accordance with current trends in behavioral decision theory, the players are assumed to act as if they form simplified representations of the game, adopting short-terms, surrogate objectives and simple heuristics in choosing strategies intended to attain the long-term objective of highest final rank.

Subsequent to the myopic and hyperopic models, which should be regarded as complementary rather than antithetical (Laing & Morrison, 1974b), Friend, Laing, and Morrison (1978) proposed and tested a two-signal model, which combines Gamson's minimum resource theory (1961) and the hyperopic model to yield

predictions concerning both coalition frequencies and payoff disbursement. Gamson's theory postulates that players entering into a coalition agreement divide their joint payoff according to a parity norm specifying that payoffs should be disbursed in proportion to the resources the players contribute to the coalition. In some experimental paradigms there is a natural way to assess the players' "resources" or "power," such as in Vinacke's experiment (1959) in which each player's power is determined by the "weight" assigned to him or her by the experiment. But in CF games there is no natural way to determine the player's power. Consequently, in its present form the two-signal model of Friend et al. (1978) is not directly applicable to our CF game experiment. However, the idea of combining two different signals in the same model is taken up in the mixed-signal model we propose below.

In a more recent study Chertkoff, Skov, and Catt (1980) tested the myopic and hyperopic models by introducing a new experimental paradigm differing from the one employed by Laing and Morrison in three major respects: (a) a sequence length that is prespecified and known to the subjects; (b) a different procedure for conducting the bargaining, which is supposed to reduce the effects of variables not included in the myopic and hyperopic models; and (c) unequal rank positions assigned to the three players when the sequence of games of status starts.

Following and extending these recent developments by Friend et al. (1978) and Chertkoff et al. (1980), the present study has two major purposes. The first is to generalize the experimental procedure without detracting from the applicability of the myopic and hyperopic models. This generalization is achieved by (a) assigning different values to the three permissible 2-person coalitions, (b) introducing dependency between trials by making the characteristic function presented on trial t dependent on the outcome of trials $1, \dots, t-1$, and (c) employing a computer-controlled bargaining procedure which completely eliminates face-to-face negotiations. The second major purpose is to develop and test a new heuristic model, called the *mixed-signal* (MS) model, which predicts both coalition frequencies and payoff allocation.

BASIC CONCEPTS

Characteristic Function Games

Negotiable coalition formation situations are frequently abstracted as games in CF form with sidepayments (Luce & Raiffa, 1957). Let N denote the set of players in the game. A *characteristic function* is a rule that assigns a real number value $v(S)$ to any $S \subseteq N$. S is called the *coalition*, and its *value*, $v(S)$, represents the reward jointly commanded by the members of coalition S against the remaining players in the game. A coalition is formed when its members agree on how to allocate its value among the members. The assigned real value function v and the rules governing communication among the players completely specify the game. It is assumed in the following that $v(\phi) = v(i) = v(N) = 0$.

To exemplify the notion of a CF, consider one of the 3-person games employed in the present study:

$$G1: \quad v(AB) = 300, \quad v(AC) = 200, \quad v(BC) = 270.$$

In game $G1$, $N = \{A, B, C\}$ and $n = |N| = 3$. If players A and B form a coalition, they jointly have 300 units of reward to disburse between themselves, leaving C with zero reward. Similarly, players A and C have 200 units to share between themselves, whereas B and C jointly command 270 units.

The outcome of a CF game is represented by a *payoff configuration* (PC), which has the form

$$(\mathbf{x}; \mathbf{S}) = (x_1, x_2, \dots, x_n; S_1, S_2, \dots, S_r).$$

A PC consists of two parts, separated from each other by a semicolon. The first part pertains to the allocation of reward among the n players, and the second to the various coalitions formed. $\mathbf{x} = (x_1, \dots, x_n)$ is an n -dimensional row vector of real numbers, called the *payoff vector*, representing a realizable allocation of payoff among the n members, who appear in alphabetical order. Thus, x_i is the payoff of player i in the allocation \mathbf{x} . $\mathbf{S} = \{S_1, \dots, S_r\}$ is a set of r mutually exclusive and collectively exhaustive coalitions ($1 \leq r \leq n$), called a *coalition structure*. Players are assumed to obtain their joint reward as specified by the CF. Thus

$$\sum_{i \in S_j} x_i = v(S_j), \quad \text{if } S_j \in \mathbf{S}.$$

To illustrate these terms, consider game $G1$. If at some stage players A and C were to consider the coalition AC with A receiving $x_A = 120$ and C receiving $x_C = 80$, leaving B with zero reward, this consideration would be denoted as the PC $(120, 0, 80; AC, B)$.

Sequential Games of Status

Suppose that the same three players participate in a sequence of temporally disjoint CF games, and denote by $x_{i,t}$ the payoff obtained by player i in the game played on trial t ($t = 1, 2, \dots$). Before the sequence starts, at trial 0, each player i gets an endowment of $x_{i,0}$. Then the total score accumulated by player i across the first t trials of the sequence is $\delta_{i,t} = \sum_{h=0}^t x_{i,h}$. Knowing the total scores of all three players at the end of any trial t , we can compute $r_{i,t}$ —the *rank* of player i 's total score relative to the scores of the other two players. Player i is said to have a higher rank than player j at the end of t trials, written as $r_{i,t} > r_{j,t}$, if and only if $\delta_{i,t} > \delta_{j,t}$. Following Laing and Morrison (1974a), the two-way tie between the two players with the highest rank is denoted by $r = 1.5$ and the two-way tie between the two players with the lowest rank by $r = 2.5$. The three-way tie is denoted by $r = 2$. There are altogether five possible ranks: 1, 1.5, 2, 2.5, 3.

A social situation falling within the domain analyzed by Laing and Morrison's models (1973) must satisfy three conditions:

- (1) The same three players participate in a sequence of one or more temporally disjoint 3-person CF games with side payments (in which $v(ABC) = 0$).
- (2) Each of the three players seeks to maximize the rank (status) he or she holds at the end of the sequence.
- (3) Each of the three players is uncertain about the number of trials in the sequence.

At the end of trial t , the attainable ranks for each subject depend both on the distribution of total scores at the end of trial $t - 1$ and the CF values, $v(S)_t$, at trial t . For example, suppose $\delta_{A,t-1} = 30$, $\delta_{B,t-1} = 20$, and $\delta_{C,t-1} = 5$. Let the CF on trial t be given by

$$G2: \quad v(AB)_t = 12, \quad v(AC)_t = 14, \quad v(BC)_t = 10.$$

Clearly, $r_{A,t-1} = 1$, $r_{B,t-1} = 2$, and $r_{C,t-1} = 3$. At the end of trial t , player B may improve her rank through coalitions AB or BC , but player B 's rank will not deteriorate if coalition AC forms. There is no way for player C to improve her rank position (even if $x_{C,t} = 14$ in coalition AC or $x_{C,t} = 10$ in coalition BC , player C is left with the third rank). Suppose, however, that the distribution of total scores at the end of trial $t - 1$ is as above, but that the CF on trial t is given by

$$G3: \quad v(AB)_t = v(AC)_t = v(BC)_t = 50.$$

Then player C may attain each of the five possible ranks. The two examples above show that to achieve a wide range of sequential 3-person games of status, where various combinations of ranks on successive trials are possible, the CF values $v(S)_t$ must be chosen judiciously.

MODELS FOR SEQUENTIAL 3-PERSON GAMES OF STATUS: BARGAINING HEURISTICS

In line with recent theoretical developments in behavioral decision theory, which postulate simplified representations of the task by the subject (Slovic, Fischhoff, & Lichtenstein, 1977), the approach proposed by Laing and Morrison for sequential games of status assumes

that players are unable to represent the overall structure of our sequential three-person game in all its complexity. In particular, we depart from game-theoretic approaches by assuming that players, lacking omniscience, act as if they form simplified representations of the game, adopting short-term, surrogate objectives and simple rules of thumb in choosing strategies intended to attain the long-run objective of highest final rank. (Laing & Morrison, 1973, p. 5).

To describe these rules of thumb, or heuristics, several terms are necessary.

Define the *interval position* of player i at the end of stage t to be the sum of differences between his or her total score and those of the other two players:

$$d_{i,t} = (\delta_{i,t} - \delta_{j,t}) + (\delta_{i,t} - \delta_{k,t}),$$

for distinct players i, j , and k . Player i 's *position* at the end of trial t is defined in terms of both the rank and interval position:

$$p_{i,t} = (r_{i,t}, d_{i,t}).$$

Bargaining heuristics or decision rules delimiting the payoff divisions between coalition partners are incorporated into three behavioral assumptions that all three players are expected to obey.

Assumption 1. A member of the winning coalition would not agree to a payoff vector that causes his or her coalition partner to overtake or pass this member in rank.

Assumption 2. A member of the winning coalition would not agree to a payoff vector that not only fails to grant him or her an improved rank but also causes the member to lose interval position.

Assumption 3. A member of the winning coalition who holds a rank-based preference for his or her partner would not accept a payoff smaller than that necessary to achieve the rank upon which this preference is based.

The first two assumptions state a version of the principle of individual rationality (Kahan & Rapoport, 1984; Rapoport, 1970); players will not enter into coalitions in such a manner as to deteriorate their overall positions. The meaning of Assumption 3 becomes clear after defining "rank-based preference" below.

Using these three assumptions, the myopic, hyperopic, and mixed-signal models described below identify a *negotiation range*—the set of all alternative agreements consistent with the three bargaining heuristics. They all predict that the actual payoff vector will lie in this range. The three assumptions above are sufficient to predict uniquely for each 2-person coalition the rank each player will attain if that coalition forms; all agreements within a negotiation range yield the same rank outcomes (Friend et al., 1978).

The Myopic Model

All the models described below assume that players forget or ignore all history of the play except as summarized by the current distribution of accumulated scores. Adhering to this assumption, as well as to Assumptions 1 through 3 above, the myopic and hyperopic are two alternative models which differ from each other in the planning horizon players are assumed to use. The myopic model (termed model M) assumes that players adopt the surrogate objective of maximizing position on the present trial, ignoring all future trials. Based on Assumptions 1 through 3 above and on the assumption that the potential coalitional partners will employ the same

reasoning as he or she, each player finds the best rank that can be achieved from each coalition and the minimum payoff needed to achieve that rank.

The best ranks for each possible 2-person coalition serve as the basis for determining which coalition the player will choose ("rank-based preference"). These choices are not deterministic, but rather are governed by a probabilistic mechanism with a single parameter ε . (The parameter ε is assumed to be fixed for all three players in the triad and for all the games in the sequences.) If player i expects to attain a higher rank from a coalition with player j rather than with k , then the myopic model assumes that i will choose j with probability $a_{ij} = 1 - \varepsilon$ and k with probability $a_{ik} = \varepsilon$, where $0 < \varepsilon < \frac{1}{2}$. According to the myopic model, each player i who is indifferent between j and k as alternative coalition partners on the basis of rank consideration, chooses j and k with probabilities in proportion to the probabilities that his or her choice is reciprocated: $a_{ij}/a_{ik} = a_{ji}/a_{ki}$. In later papers (Friend, Laing, & Morrison, 1977, 1978) the assumption is simply that a rank-indifferent player chooses each possible partner with probability $\frac{1}{2}$. We adopted the earlier assumption because it better fits our data.

The probabilistic preferences constitute the *attraction structure* of the game, which is used, in turn, to generate the probabilities of each coalition forming, such that a 2-person coalition will form with a (normalized) probability equal to the product of the players' preferences for each other. For example, if player A prefers player C , C prefers B , and B is indifferent between A and C (based on rank consideration), then the attraction structure has player A choosing B and C with probabilities $a_{AB} = \varepsilon$ and $a_{AC} = 1 - \varepsilon$, player C choosing A and B with probabilities $a_{CA} = \varepsilon$ and $a_{CB} = 1 - \varepsilon$, and player B choosing A and C with probabilities $a_{BA} = \varepsilon$ and $a_{BC} = 1 - \varepsilon$ (the reciprocity assumption). Therefore, the coalitional probabilities $p'(ij)$ are

$$p'(AB) = \varepsilon^2, \quad p'(AC) = \varepsilon(1 - \varepsilon), \quad p'(BC) = (1 - \varepsilon)^2. \quad (1)$$

The probability that no coalition is formed is $1 - p'(AB) - p'(AC) - p'(BC)$. In a manner similar to Chertkoff (1967), it is assumed that when no coalition is formed, negotiations begin afresh; therefore, the normalized coalition probabilities are obtained by dividing each of the unnormalized probabilities by their sum. In the example above, $p'(AB) + p'(AC) + p'(BC) = \varepsilon^2 - \varepsilon + 1$, so that each probability in Eq. (1) should be divided by that amount to obtain the normalized probabilities, $p(ij)$.

The myopic model predicts that the payoff vector will lie in the negotiation range identified on the basis of the three bargaining heuristics. It further asserts that any disagreement between coalition partners over alternative allocations within the negotiation range will be resolved in favor of the player who enjoys a bargaining advantage within the coalition. Within a coalition ij , player i enjoys a *bargaining advantage* over j if and only if $a_{ij}/a_{ki} < a_{ji}/a_{kj}$. The model "predicts that the payoff to the player enjoying a bargaining advantage over his partner will tend to lie toward the former's preferred end of the negotiation range" (Friend et al., 1978, p.

33). If no bargaining advantage is identified within the coalition, then the partners will tend to agree to that payoff allocation nearest the mid-point of the negotiation range.

To exemplify the predictions of the myopic model, consider a situation where the total scores at the end of trial $t-1$ are

$$\delta_{A,t-1} = 160, \quad \delta_{B,t-1} = 200, \quad \delta_{C,t-1} = 130,$$

and the characteristic function on trial t is

$$v(AB)_t = 90, \quad v(AC)_t = 80, \quad v(BC)_t = 70.$$

Clearly, $r_{A,t-1} = 2$, $r_{B,t-1} = 1$, and $r_{C,t-1} = 3$. Player A expects to achieve sole possession of first place through coalition AC but to remain in her current position if coalition AB forms. Thus, she has rank-based preference for coalition AC , which means that she chooses C with probability $a_{AC} = 1 - \varepsilon$ and B with $a_{AB} = \varepsilon$. Player C prefers coalition BC over coalition AC for a similar reason, choosing B with probability $a_{CB} = 1 - \varepsilon$ and A with probability $a_{CA} = \varepsilon$. Expecting to remain in her first position, player B is indifferent between her prospective coalition partners (on rank consideration). Hence, in accordance with the reciprocity assumption $a_{BA} = \varepsilon$ and $a_{BC} = 1 - \varepsilon$.

The attraction structure is used next to determine bargaining advantages. Player B enjoys a bargaining advantage over A because $a_{BA}/a_{CB} < a_{AB}/a_{CA}$. Player C has a bargaining advantage over A because $a_{CA}/a_{BC} < a_{AC}/a_{BA}$. And player C also enjoys a bargaining advantage over B because $a_{CB}/a_{AC} < a_{BC}/a_{AB}$.

Consider first coalition AB . As player B is first-ranked and A is second-ranked, B will not accept any payoff allocation that reverses their rank order, so a preliminary boundary on any agreement between them is $x_{A,t} - x_{B,t} < 40$ (Assumption 1). But this boundary is superceded for this coalition by Assumption 2, which states that if there is no improvement in rank order, then a player will not accept an offer that would lose her interval position. To maintain interval position, each player must receive at least an outcome of 30, so the negotiation range is between $(30, 60; AB)$ and $(60, 30; AB)$. Because player B enjoys a bargaining advantage over A , it is predicted that the payoff to B will tend to lie toward her preferred end of the negotiation range ("tend to lie" has been interpreted to mean that the payoff will fall above the mid-point of the negotiation range), namely, $(90 - x_B, 45 < x_B < 60; AB)$.

Similar considerations applied to coalitions AC and BC result in the following set of PCs:

$$\begin{aligned} (90 - x_B, 45 < x_B \leq 60, 0; AB, C) \\ (80 - x_C, 0, 33\frac{1}{3} < x_C \leq 40; AC, B) \\ (0, 70 - x_C, 38\frac{1}{3} < x_C \leq 46\frac{2}{3}; BC, A). \end{aligned}$$

The coalition structure probabilities are, respectively,

$$\frac{\varepsilon^2}{\varepsilon^2 - \varepsilon + 1}, \quad \frac{(1 - \varepsilon)\varepsilon}{\varepsilon^2 - \varepsilon + 1}, \quad \frac{(1 - \varepsilon)^2}{\varepsilon^2 - \varepsilon + 1}.$$

They can be determined numerically once ε is estimated from the data.

The Hyperopic Model

Under the assumptions of the hyperopic model, termed model *H*, players behave as if whatever coalition structure forms in the present trial will continue to form in subsequent trials. This viewpoint does not presume that the payoff disbursement within that coalition structure will not change, nor does it explicitly call for a multiple-game agreement. Rather, the actual coalition values $v(S)$ are ignored and, instead, the relative standings of the players are the focal points of negotiations.

The effect of looking indefinitely into the future is that if there exists a difference in ranks between two members of a coalition, it can never be overcome. However, any disadvantage in rank of a player inside the coalition to a player outside the coalition can always be overcome. This viewpoint determines the coalitional preferences, which are then transformed into the probabilities a_{ij} as in the myopic model. In the limit, there are only four distinct rank-order structures among three players, each of which leads to a unique attraction structure (Table 1). The point disbursements for given coalition structures are determined exactly as in the myopic model; presumably the point allocation, if not the coalition structure, is renegotiated on subsequent trials.

The Mixed-Signal Model

In reflecting on the results of experiments on weighted majority games (Shapley, 1962), Friend et al. (1978) noted that both relative status and the players' resources

TABLE I
Hyperopic Limiting Attraction Structure Probabilities

Condition	Rank-order structure			
	$A = B = C$	$A = B > C$	$A > B = C$	$A > B > C$
$p(A \text{ chooses } B)$.50	ε	.50	.50
$p(A \text{ chooses } C)$.50	$1 - \varepsilon$.50	.50
$p(B \text{ chooses } A)$.50	ε	ε	ε
$p(B \text{ chooses } C)$.50	$1 - \varepsilon$	$1 - \varepsilon$	$1 - \varepsilon$
$p(C \text{ chooses } A)$.50	.50	ε	.50
$p(C \text{ chooses } B)$.50	.50	$1 - \varepsilon$.50
$p(AB)$	$\frac{1}{3}$	$\varepsilon^2/(1 - \varepsilon + \varepsilon^2)$	$(\varepsilon/2)/(1 - \varepsilon + \varepsilon^2)$	$\frac{2}{3}\varepsilon$
$p(AC)$	$\frac{1}{3}$	$[(1 - \varepsilon)/2]/(1 - \varepsilon + \varepsilon^2)$	$(\varepsilon/2)/(1 - \varepsilon + \varepsilon^2)$	$\frac{1}{3}$
$p(BC)$	$\frac{1}{3}$	$[(1 - \varepsilon)/2]/(1 - \varepsilon + \varepsilon^2)$	$(1 - \varepsilon)^2/(1 - \varepsilon + \varepsilon^2)$	$\frac{2}{3}(1 - \varepsilon)$

(weights) were attended to by the subjects. To account for these data, they proposed a two-signal model based on the assumption that "players in complex coalition situations do attend to more than one relevant signal in their decision environment, and these signals do act jointly to influence alliances and negotiated payoffs within those alliances" (1978, p. 23). The term "signal" in their theory denotes any cue which may be used by players in their decisions. Friend et al. (1978) provided evidence in support of the model suggesting that subjects may adopt the logical combination of two one-signal models as a simplifying heuristic to cope with the complexity of the bargaining situation.

In a similar manner the MS model assumes that the sequential 3-person CF game of status in which the restriction $v(AB)_i = v(AC)_i = v(BC)_i$ no longer holds generates two prominent and distinct signals, each of which determines a different attraction structure. The first is the status signal as identified by the myopic model; the second signal is the CF values. Our main assumption (based on experiments on ordinary CF games) concerns the second signal: each player prefers to join the coalition with the highest value. Just as subjects who are asked to maximize their absolute gain are, nevertheless, concerned with relative gain, so subjects who are asked to maximize relative gain are concerned with their absolute gain.

Before applying the MS model to data obtained from status games, we first test the effectiveness of the second signal in a class of games in which it should be most important, namely, the single-stage CF game with side payments in which subjects are paid in proportion to their absolute scores. The CF values should be the sole signal in this class of games.

Parameter estimation. Assuming that the preferred partner is chosen with probability $1 - \varepsilon$ and the other partner with probability ε ($0 \leq \varepsilon < \frac{1}{2}$), Table 2 presents the four distinct attraction structures based on the four ordinal relations between the three coalition values $v(AB)$, $v(AC)$, and $v(BC)$.

TABLE 2
Attraction Structure Probabilities for 3-Person Characteristic Function Games

Condition	Rank-order structure			
	$v(AB) = v(AC) = v(BC)$	$v(AB) > v(AC) = v(BC)$	$v(AB) = v(AC) > v(BC)$	$v(AB) > v(AC) > v(BC)$
$p(A \text{ chooses } B)$.50	$1 - \varepsilon$.50	$1 - \varepsilon$
$p(A \text{ chooses } C)$.50	ε	.50	ε
$p(B \text{ chooses } A)$.50	$1 - \varepsilon$	$1 - \varepsilon$	$1 - \varepsilon$
$p(B \text{ chooses } C)$.50	ε	ε	ε
$p(C \text{ chooses } A)$.50	.50	$1 - \varepsilon$	$1 - \varepsilon$
$p(C \text{ chooses } B)$.50	.50	ε	ε
$p(AB)$	$\frac{1}{3}$	$(1 - \varepsilon)^2 / (1 - \varepsilon + \varepsilon^2)$	$[(1 - \varepsilon)/2] / (1 - \varepsilon + \varepsilon^2)$	$(1 - \varepsilon)^2 / (1 - \varepsilon + \varepsilon^2)$
$p(AC)$	$\frac{1}{3}$	$(\varepsilon/2) / (1 - \varepsilon + \varepsilon^2)$	$[(1 - \varepsilon)/2] / (1 - \varepsilon + \varepsilon^2)$	$\varepsilon(1 - \varepsilon) / (1 - \varepsilon + \varepsilon^2)$
$p(BC)$	$\frac{1}{3}$	$(\varepsilon/2) / (1 - \varepsilon + \varepsilon^2)$	$\varepsilon^2 / (1 - \varepsilon + \varepsilon^2)$	$\varepsilon^2 / (1 - \varepsilon + \varepsilon^2)$

The parameter ε in Table 2 may be estimated and then used to generate numerical predictions for $p(ij)$. When $v(AB) > v(AC) > v(BC)$, as is the case in all of the four studies that we examine below, the likelihood function is given by

$$L = \frac{(f_{AB} + f_{AC} + f_{BC})!}{(f_{AB})! (f_{AC})! (f_{BC})!} \cdot \left[\frac{(1 - \varepsilon)^2}{1 - \varepsilon + \varepsilon^2} \right]^{f_{AB}} \cdot \left[\frac{\varepsilon(1 - \varepsilon)}{1 - \varepsilon + \varepsilon^2} \right]^{f_{AC}} \cdot \left[\frac{\varepsilon^2}{1 - \varepsilon + \varepsilon^2} \right]^{f_{BC}}$$

where f_{ij} is the observed frequency of coalition ij . Taking the logarithm of L , differentiating $\log L$ with respect to ε , setting the result equal to zero, and solving for ε , yields the maximum likelihood estimate

$$\hat{\varepsilon}_{1.2} = \frac{(f_{AB} + 2f_{AC} + 3f_{BC}) \pm \sqrt{(f_{AB} + 2f_{AC} + 3f_{BC})^2 + 4(f_{AB} - f_{BC})(f_{AC} + 2f_{BC})}}{2(f_{BC} - f_{AB})}. \quad (2)$$

Maximum likelihood estimates may also be obtained for the two other nontrivial cases: $v(AB) > v(AC) = v(BC)$ and $v(AB) = v(AC) > v(BC)$.

An experimental test. Table 3 presents observed and predicted coalition frequencies for four different 3-person game experiments in which the coalition values were rearranged so that $v(AB) > v(AC) > v(BC)$. The first study is due to Riker (1967), who had three different groups of subjects, consisting of businessmen and undergraduate students, play the following 3-person CF game once:

$$v(AB) = \$6.00, \quad v(AC) = \$5.00, \quad v(BC) = \$4.00.$$

Of the 93 plays, 2-person coalitions were formed in 90 cases, whereas in 3 cases no agreement was reached. The second study is due to Kahan and Rapoport (1974), who had three groups of subjects under three different communication conditions. There were four triads in each group: each triad played five different CF games that

TABLE 3
Observed and Predicted Coalition Frequencies for
Four 3-Person Characteristic Function Game Studies

Study		Coalition			Total	$\hat{\varepsilon}$	χ^2
		AB	AC	BC			
Riker (1967)	Observed	44	28	10	90	0.339	0.0005
	Predicted	44.94	28.09	17.96	90		
Kahan & Rapoport (1974)	Observed	146	52	41	239	0.329	4.098
	Predicted	138.09	67.71	33.20	239		
Medlin (1976)	Observed	49	23	17	89	0.362	0.817
	Predicted	47.11	26.73	15.17	89		
Levinsohn & Rapoport (1978)	Observed	112	45	23	180	0.305	0.411
	Predicted	110.33	48.42	21.25	180		

were repeated four times. Table 3 summarizes the coalition frequencies over the three communication conditions and the five games. Medlin (1976) also presented his subjects with five different 3-person CF games. There were four experimental conditions in his study differing from one another in the value of $v(N)$. Of a total of 160 plays, the grand coalition formed on 71 plays. Table 3 reports only the frequencies of plays terminating with the formation of a 2-person coalition summed over games and conditions. The fourth study by Levinsohn and Rapoport (1978) included a variety of 3-person CF games with no grand coalition. One condition included 90 single-stage games, and a second condition consisted of 18 multi-stage games each of which played for five trials. Table 3 summarizes the coalition frequencies over games and both experimental conditions.

Table 3 provides strong support for the probability model of coalition formation in 3-person CF games. In all four studies Eq. (2) yields $0 \leq \hat{\varepsilon} < \frac{1}{2}$ as expected. Moreover, despite the wide variety of CF games and the procedural differences among the four studies, the range of $\hat{\varepsilon}$ is quite narrow, from 0.305 in the study by Levinsohn and Rapoport (1978) to 0.390 in the study by Riker (1967). As shown in Table 3, χ^2 is nonsignificant for each of the four studies.

We have, then two kinds of models, one focusing on status, and the other on coalition values. We have chosen the myopic model as one component of the MS model because of its relative success in previous experiments (Chertkoff et al., 1980). Each model predicts an attraction structure with a single parameter ε from which the coalition frequencies and the payoff disbursement can be derived. We must next consider how players' perceptions of the two signals interact with each other in influencing the overall pattern of outcomes. Model MS assumes that the two signals are combined with equal weights. This assumption breaks down into several cases. Let a'_{ij} and a''_{ij} denote the probability that player i will choose j according to the M and CF models, respectively. Denote by a_{ij} the same probability according to the MS model. Then

$$\begin{aligned}
 & a_{ij} = \varepsilon, \quad \text{if } a'_{ij} = \varepsilon \text{ and } a''_{ij} = \varepsilon, \\
 & \text{or } a'_{ij} = \frac{1}{2} \text{ and } a''_{ij} = \varepsilon, \quad \text{or } a'_{ij} = \varepsilon \text{ and } a''_{ij} = \frac{1}{2}; \\
 & a_{ij} = \frac{1}{2}, \quad \text{if } a'_{ij} = \varepsilon \text{ and } a''_{ij} = 1 - \varepsilon, \\
 & \text{or } a'_{ij} = 1 - \varepsilon \text{ and } a''_{ij} = \varepsilon, \quad \text{or } a'_{ij} = a''_{ij} = \frac{1}{2}; \\
 & a_{ij} = 1 - \varepsilon, \quad \text{if } a'_{ij} = 1 - \varepsilon \text{ and } a''_{ij} = 1 - \varepsilon, \\
 & \text{or } a'_{ij} = \frac{1}{2} \text{ and } a''_{ij} = 1 - \varepsilon, \quad \text{or } a'_{ij} = 1 - \varepsilon \text{ and } a''_{ij} = \frac{1}{2}.
 \end{aligned}$$

Once the attraction structure is specified, the MS model determines who has the bargaining advantage and subsequently the payoff vector with the negotiation range in exactly the same way as do the myopic and hyperopic models.

PREVIOUS AND CURRENT EXPERIMENTAL PROCEDURES

In surveying experimental research on coalition forming behavior, Kahan and Rapoport noted that "creating an experimental game to study coalition formation behavior is no straightforward task, and virtually every investigator has his own idiosyncratic approach" (1984, p. 249). When several methodologies are employed, problems of comparison and integration arise, because the subjects' interpretations of the task, their motivations, their aspirations, and consequently the ensuing negotiations are affected by the experimental formulation of the task. Komorita and Meek (1978), for example, have demonstrated that the same experimental game supports differing social psychological theories, depending on the experimental conditions of information and communication. Similar procedural effects may take place in games of status. The present experiment differs from previous experimental studies of 3-person games of status in three important respects. The reasons for these changes are described in some detail below.

In the experiments of Laing and Morrison (1973, 1974a, 1974b), bargaining was face-to-face, completely informal, supervised, and public. In terms of generality, clarity of the task, integrity of the coalition formation process, and flexibility of negotiations, the unrestricted bargaining approach is an excellent technique. However, the control of motivation poses an obvious problem because face-to-face communication enhances the chances that the players' judgments will be altered by personality characteristics of their various opponents instead of the (for purposes of testing theory) structural variables imposed by the nature of the game. A vivid description of unrestricted negotiations has been provided by Kalisch, Milnor, Nash, and Nering:

The tendency of a player to get into coalitions seemed to have a high correlation with talkativeness.... In many cases, aggressiveness played a role even in the first formation of a coalition; and who yelled first and loudest after the umpire said 'go' made a difference in the outcome. (1954, p. 307)

To resolve this problem, Chertkoff et al. had their subjects make initial choices concerning coalition partners in private. However, once two players chose each other, the negotiations of the terms of the agreement were conducted by the two coalition members face-to-face. Consequently, the procedure of Chertkoff et al. reduces, but certainly does not eliminate "the impact of variables not included in the Laing and Morrison theories" (Chertkoff et al., 1980, p. 254). In contrast, the present study employs the computer-controlled experimental paradigm NPER (Kahan & Helwig, 1971; Kahan & Rapoport, 1984; Rapoport & Kahan, 1974), which eliminates the effects of face-to-face bargaining. Details of the procedure are presented in the method section below.

In the Laing and Morrison experiments, the three players started the sequence with point totals $x_{A,0} = x_{B,0} = x_{C,0} = 0$, and then participated in a series of 3-person games of unknown length with

$$v(AB)_t = v(AC)_t = v(BC)_t = k, \quad k = 100, 300, 500.$$

As noted by Chertkoff et al., this procedure gives rise to a limited number of attraction structures the frequency of which cannot be controlled. It is, therefore, not always possible to reach conclusions about coalition forming behavior in prespecified attraction structures of special theoretical interest. In Chertkoff et al., players draw from a box three different cards with initial point $x_{A,0} \neq x_{B,0} \neq x_{C,0} > 0$, thus guaranteeing at least four different kinds of attraction structures on trial 1. However, the attraction structures are no longer manipulatable after the first trial because of the constraint $v(AB)_i = v(AC)_i = v(BC)_i = 100$. In contrast, the present study employs a procedure in which the CFs differ from one trial to another and the coalition values differ, in general, from one another. It was argued above that this procedural change is of sufficient importance to introduce a new prominent signal into the coalition formation process and to control the attraction structures throughout the sequence.

The third major procedural difference between the present study and its predecessors concerns sequential dependencies in the CFs. In all previous studies of sequential games of status the CFs within a sequence were mutually independent. But if sequential games of status are intended to model or at least to reflect the major characteristics of continuing social interactions (Laing & Morrison, 1973), sequential dependencies between CFs may not be ignored. Rather, they ought to be incorporated into the experimental task (Levinsohn & Rapoport, 1978). In the present study, the coalition values $v(AB)_i$, $v(AC)_i$, and $v(BC)_i$ depend on the outcomes of the previous trials, as described in the method section below.

A 3-PERSON GAME EXPERIMENT

Method

Subjects. Subjects were 25 male and female volunteers, mostly undergraduate students at the University of Haifa, Israel, who were offered the opportunity to earn cash for their performance in a multi-session coalition formation experiment. To save time on instructions and training with the experimental procedure, only volunteers who had previously participated in a four session 12-hr coalition formation experiment employing the same computer-controlled *Coalitions* paradigm (see below) were recruited. This previous experiment had been designed to study coalition formation and payoff disbursement in 4-person CF games. Unlike the present study, the previous games had been sequentially independent with the identity of the subjects being rotated randomly from one game to another.

Coalitions. The present experiment employed the *Coalitions* package of programs. A full description of the computer program can be found in Kahan, Coston, Helwig, Rapoport, and Wallsten (1976), Kahan and Rapoport (1984), and numerous other publications; only a brief description is presented here.

Coalitions defines bargaining as passing through three stages, which loosely correspond to the four stages of bargaining of Thibaut and Kelley (1959). The first

TABLE 4
Key Words and Their Effects

Keyword	Example of parameter	Primary effect	Other effect on this coalition	Effects outside named coalition
PASS	None	Send null message	None	None
SOLO	None	Immediate ratification of player's 1-person coalition	None	Player is removed from further play in this game
OFFER	$A = 20, B = 18$	Make a proposal	Supersedes player's previous OFFER, AGREE, and ACCEPT	None
SUGGEST	$A = 20, C = 15$ to AC	Make informal, secret proposal	None	None
REJECT	AC BY C	Erase an offer	None	None
AGREE	AC BY C	Show liking for an offer	Same as for OFFER	Erases any ACCEPT made by this player
ACCEPT	AC BY C	Tentative commitment to an offer	Same as for OFFER	Erases any AGREE or ACCEPT made by this player
When all players have accepted an offer		Game moves to acceptance stage	None	Erases all other positions taken by included players (they may make no new OFFER except to subsets or supersets of the accepted coalition)
AFFIRM	AC	Stand by previous action	None	None
RATIFY	None	Propose move to ratification	Form coalition if all members agree to ratification (coalition is dissolved otherwise)	Concludes game for players in ratified coalitions
MESSAGE VALUE	$A = 1, C = 3$ None	Send numbered message(s) to named player(s) (counts as communication) Information: Give all values in the game except those involving removed players (does not count as a communication)		

stage, called the *negotiation stage*, corresponds to Thibaut and Kelley's sampling and bargaining phases of the process, in which players first learn of the available outcomes and then establish bargaining positions through negotiations and proposals to each other. The second, or *acceptance stage*, corresponds to Thibaut and Kelley's commitment phase. Players comprising a coalition have tentatively agreed upon a particular allocation (PC) of their joint payoff. Although this agreement is not yet binding, it is sufficiently strong to restrict players' bargaining mobility. Players not in the agreement attempt to disrupt it in favor of a PC more favorable to themselves, while those in the coalition must decide on maintenance of the present agreement versus gambling for a better outcome within or outside of that coalition. The game then passes to the third or *ratification stage*, which corresponds to Thibaut and Kelley's institutionalization phase. When an accepted PC, having survived for a certain length of time, is voted into ratification by its members, the game terminates and each player is given his or her payoff as prescribed by the ratified PC.

Subjects never learn the personal identity of their co-players. They communicate with each other by transmitting messages coded in keywords via CRTs connected to a computer. The computer checks the legality of the messages, reformats them to be easily readable, adds messages informing players of the effects of the present move on previous moves in the game, and transmits the entire package to its intended recipients. The keywords used by *Coalitions* and their effects, as well as the

TABLE 5

List of Messages (Translated from Hebrew) for Games of Status

-
1. Believe me, for Pete's sake.
 2. My next offer is serious.
 3. If you accept my offer, you will improve your rank.
 4. Join me to decrease your point discrepancy with the other player.
 5. Join me now, as this may be the final game.
 6. Join me in coalition for all the remaining games in the present sequence.
 7. If you accept my offer, I'll join you in coalition for the remaining games.
 8. I'll join you in coalition for all the remaining games.
 9. If you don't accept my offer, you will never pass the other player.
 10. My offer gives you more points.
 11. Make a reasonable offer and we'll join together.
 12. You are too greedy.
 13. If we stick together, our situation will improve.
 14. Don't trust the other player, stay with me.
 15. I refuse to change my position.
 16. Decide, and I'll follow suit.
 17. Let us wait before ratification to see what happens.
 18. Let us ratify as soon as possible.
 19. I may terminate the game with a "solo."
 20. I agree with your last message sent to me.
-

implications of being in the acceptance stage (called "All-accepted" in the table) are explained in Table 4.

Communication. A chart of keywords that were always available to the subjects is shown in Table 4. The keyword MESSAGE, which counts as a communication, allows the subject to send a prespecified message by only typing its number. Based on extensive pretests, a list of 20 common messages was specifically prepared for the present study; it is presented in Table 5.

Experimental procedure. Twenty-three experimental sessions of approximately 3 hr each were conducted. In each session, 3 subjects were randomly selected from the group of 25 and assigned to the session under the constraint that no 2 players participated together in more than one session. Each session consisted of two sequences of 3-person games of status. Independence between sequences was achieved by randomly rotating the roles of the three players from one sequence to another. Within a sequence the three players maintained their roles. Forty-six sequences were generated, each including between 1 and 11 games. Altogether, 263 3-person CF games were played. The number of games per sequence varied considerably from one sequence to another; it was predetermined but not disclosed to the subjects.

Upon arrival in the laboratory, each subject was reacquainted with the experimental apparatus. The rules of the game of status were then introduced in a brief session consisting of written instructions and verbal elaborations. First, the *Coalitions* program was briefly reviewed. Then the specific features of games of status were emphasized. The subject was told that he or she would participate in several sequences of 3-person CF games of different length, in which the same roles would be maintained over all games. When a sequence terminated after a prespecified but unknown number of trials, the members of the triad were paid \$5.00, \$3.00, \$2.00, \$1.00, and \$0.50 for ranks 1, 1.5, 2, 2.5, and 3, respectively. Additionally, \$0.50 was paid to each subject for each hour of play. Each subject had a full record of his or her position as well as the position of the other players during the entire sequence.

The three members of each triad were labeled *A*, *B*, or *C*. Communication on trial 1 proceed in the order *A*, *B*, *C*, *A*,.... Starting on trial 2, the order of communication was inversely related to rank, with the low player making the first move and the top player moving third. In case of a tie, the communication order was determined randomly.

Experimental games. The various CF games in each sequence were constructed to achieve two goals: (a) to reward players with a high rank on trial $t - 1$ by rendering the CF on trial t more favorable to them, and (b) to maximize the discriminability among the three models. To achieve the former goal, three transformation rules were employed to determine the *ordinal* relations between the coalition values on trial t as a function of the accumulated score at the end of trial $t - 1$:

- (1) If $\delta_{i,t-1} > \delta_{j,t-1} > \delta_{k,t-1}$, then $v(ij)_t > v(ik)_t > v(jk)_t$.
- (2) If $\delta_{i,t-1} = \delta_{j,t-1} > \delta_{k,t-1}$, then
- (a) $v(ij)_t > v(ik)_t > v(jk)_t$, if $x_{i,t-1} > 0$ and $x_{j,t-1} = 0$;
 - (b) $v(ij)_t > v(jk)_t > v(ik)_t$, if $x_{j,t-1} > 0$ and $x_{i,t-1} = 0$;
 - (c) either (a) or (b) above, if both $x_{i,t-1} > 0$ and $x_{j,t-1} > 0$ (determined randomly).
- (3) If $\delta_{i,t-1} > \delta_{j,t-1} = \delta_{k,t-1}$, then
- (a) $v(ij)_t > v(ik)_t > v(jk)_t$, if $x_{j,t-1} > 0$ and $x_{k,t-1} = 0$;
 - (b) $v(ik)_t > v(ij)_t > v(jk)_t$, if $x_{k,t-1} > 0$ and $x_{j,t-1} = 0$;
 - (c) either (a) or (b) above, if both $x_{j,t-1} > 0$ and $x_{k,t-1} > 0$ (determined randomly).

The transformation rules (1), (2), and (3) above place constraints on the CF values on trial t , but do not determine them uniquely. Because model M , H , and MS differ from one another in the attraction structures they predict, CF values for

TABLE 6
Attraction Structures for Coalition Partners by Model

Class of attraction structure	Model			Example
	Myopic M	Hyperopic H	Mixed signal MS	
I	$i \rightarrow k$ $j \rightarrow \text{equal}$ $k \rightarrow i$	$i \rightarrow \text{equal}$	$i \rightarrow \text{equal}$ $j \rightarrow i$ $k \rightarrow i$	$\delta_{i,t-1} = 104; \delta_{j,t-1} = 72; \delta_{k,t-1} = 60$ $v(ij)_t = 54; v(ik)_t = 48; v(jk)_t = 30$
II	$i \rightarrow k$ $j \rightarrow k$ $k \rightarrow i$	$j \rightarrow k$	$i \rightarrow \text{equal}$ $j \rightarrow \text{equal}$ $k \rightarrow i$	$\delta_{i,t-1} = 72; \delta_{j,t-1} = 56; \delta_{k,t-1} = 22$ $v(ij)_t = 64; v(ik)_t = 54; v(jk)_t = 26$
III	$i \rightarrow \text{equal}$ $j \rightarrow k$ $k \rightarrow j$	$k \rightarrow \text{equal}$	$i \rightarrow j$ $j \rightarrow \text{equal}$ $k \rightarrow \text{equal}$	$\delta_{i,t-1} = 74; \delta_{j,t-1} = 72; \delta_{k,t-1} = 24$ $v(ij)_t = 100; v(ik)_t = 60; v(jk)_t = 56$
IV	$i \rightarrow k$ $j \rightarrow k$ $k \rightarrow \text{equal}$	$i \rightarrow k$ $j \rightarrow k$ $k \rightarrow \text{equal}$	$i \rightarrow \text{equal}$ $j \rightarrow \text{equal}$ $k \rightarrow i$	$\delta_{i,t-1} = 90; \delta_{j,t-1} = 90; \delta_{k,t-1} = 60$ $v(ij)_t = 50; v(ik)_t = 40; v(jk)_t = 34$
V	$i \rightarrow \text{equal}$ $j \rightarrow i$ $k \rightarrow i$	$i \rightarrow \text{equal}$ $j \rightarrow k$ $k \rightarrow j$	$i \rightarrow j$ $j \rightarrow i$ $k \rightarrow i$	$\delta_{i,t-1} = 92; \delta_{j,t-1} = 56; \delta_{k,t-1} = 56$ $v(ij)_t = 66; v(ik)_t = 56; v(jk)_t = 33$

Note. $(i \rightarrow j)$ means that player i prefers to form a coalition with player j and will choose j with probability $1 - \varepsilon$ and k with probability ε ; $(i \rightarrow \text{equal})$ means that player i is indifferent between players j and k as a possible coalition partner on pure rank considerations.

trial t were chosen within the constraints imposed by the transformation rules to generate various CFs such that the predictions of the three models are maximally discriminable. Specifically, by solving sets of linear inequalities, CF values were determined for trial t , which generate one of the five classes of attraction structures (three classes according to the hyperopic model) presented in Table 6. All of the five (three) classes were represented in the experiment, though not with equal frequencies.

Table 6 shows the attraction structures predicted by the three models for the different classes we constructed. The predicted probabilities of coalitions ij , ik , and jk are easily calculated from the information in Table 6.

RESULTS

The myopic, hyperopic, and mixed-signal models provide alternative explanations of coalition behavior in 3-person games of status played in a sequence. Because each model attempts to account for the interdependence of games within a given sequence, we adopt the assumption that each game "is an 'independent event,' except for the interdependence identified by the model under investigation" (Laing & Morrison, 1973, p. 17). This assumption is not tenable if the same coalition that formed in m ($m \geq 1$) consecutive games is dictated under the terms of a single agreement; the more appropriate and meaningful unit of analysis in this case is the composite event spanning m games. As in Laing and Morrison (1973), only outcomes dictated under the terms of a single-trial agreement are, therefore, examined in the present study.

Two alternative criteria were employed to identify games falling under single-trial agreements. The first criterion (I) is based on analysis of formal communications, whereas the second criterion (II) is based on the analysis of outcomes of successive games. Under Criterion I, a sequence of games ($t, t+1, \dots, t+m$) was classified as falling under a multiple-trial agreement if at least one of the messages 6, 7, or 8 in Table 5 (all pertaining to the remaining games in a sequence) was transmitted between the members of the winning coalition at trial t (assume it is coalition ij), and in successive games (on trials $t+1, t+2, \dots, t+m$) players i and j formed a coalition while bargaining with each other with no further reference to player k . Forty-seven games were thus classified by Criterion I and removed from further analyses. Under Criterion II, a game was classified as falling under a multiple-trial agreement if it was included in a sequence of games in which (a) the same coalition was formed on each game and (b) no negotiation was conducted with the third player. One hundred twenty-four games were thus classified by Criterion II and removed from further analyses. Another game was omitted because of an error in the CF. In total, 188 and 111 games were analyzed under Criteria I and II, respectively. As the two criteria yielded very similar results, only the analyses of the games coded as single-trial agreements by Criterion I is presented below.

Tests of Bargaining Heuristics

Three bargaining heuristics, stated as Assumptions 1 through 3 above, are shared by the myopic, hyperopic, and mixed-signal models. Assumption 1 is the simplest and most straightforward, postulating that a member of a winning coalition will not accept a PC that allows his or her coalition partner to overtake or pass this member in rank. The first column in Table 7 shows the proportion of games in which this assumption was violated. The numerator shows the number of violations and the denominator the number of games in which the assumption was testable. Of the 19 games in which Assumption 1 was violated, in 2 games the two coalition members switched their ranks, in 3 games the two coalition members ended the game with equal ranks after they had started with different ranks, and in 14 games the two coalition members, having been tied for rank, ended the game with different ranks. Thus, most of the violations of Assumption 1 occurred because equality in rank was not maintained.

According to Assumption 2, a member of a winning coalition would not accept a PC that not only fails to grant him or her an improved rank but also worsens this member's interval position. Assumption 2 places stronger demands on the player than Assumption 1, because it concerns not only differences in rank, which are apparent and obvious, but also interval positions, which require nontrivial calculations. In testing Assumption 2, we omitted games in which (i) both coalition members changed their ranks, (ii) a coalition was formed between players tied in rank, or (iii) Assumption 1 was violated. The second column in Table 7 shows the proportion of games in which Assumption 2 was violated.

Assumption 3 concerns coalitions between players who hold rank-based preferences for each other. It postulates that a member of such a coalition would not accept a payoff smaller than that necessary to achieve the rank upon which this preference is based. Table 7 shows that this assumption was violated in 27 % of the games in which it was tested.

The negotiation range is the set of PCs consistent with the three bargaining heuristics considered jointly. All three models predict that the payoff vectors will fall within the negotiation range. The right-hand column of Table 7 shows that this prediction was confirmed in 61 % of all the games under single-trial agreements (67 % under Criterion II). These results are comparable to those of Laing and

TABLE 7
Proportion of Violations of the Three Bargaining Heuristics

	Assumption			
	1	2	3	1-3
Number	19/163	37/137	24/89	74/188
Proportion	.117	.270	.270	.390

Morrison, who reported that "approximately two-thirds (51/76) of the payoff allocations observed under single-trial agreements conform to this prediction" (1973, p. 18).

Coalition Frequencies

Models M, H, and MS differ from one another with respect to the predicted attraction structure. To estimate ε , a maximum likelihood procedure was employed. Let a denote a particular coalition structure, c denote a permissible 2-person coalition (ij , ik , or jk), $f(a, c)$ denote the number of games in which coalition c was formed under attraction structure a , and $P^*(a, c, \varepsilon)$ denote the probability stated by the model under consideration as a function of ε that coalition c forms under attraction structure a . Then the likelihood function is given by (Laing & Morrison, 1973)

$$L = \prod_a C_a \left[\prod_c P^*(a, c, \varepsilon)^{f(a, c)} \right], \quad (3)$$

where $C_a = [\sum_c f(a, c)]! / \prod_c [f(a, c)!]$.

Searching over the interval $[0, 1]$, the value of ε that maximizes (3) was estimated separately for each of the three models. The result was 0.44, 0.66, and 0.22 for models M, H, MS, respectively. If the players always adhere to the model's assumptions, then $\hat{\varepsilon}$ approaches zero. The estimated parameter value, $\hat{\varepsilon}$, may be used, then, as a measure of goodness of fit: the lower the value, the better the model describes the attraction structure. Using this criterion, the results support model MS over the other two models. In particular, they reject model H for which $\frac{1}{2} < \hat{\varepsilon}$, contrary to the assumption that $0 < \varepsilon < \frac{1}{2}$.

Using the estimated parameter values $\hat{\varepsilon}$, the value of the likelihood function L , denoted by L^* , was computed separately for each model. The number in each cell of Table 8 shows the resulting likelihood ratio $L_{\text{column}}^* / L_{\text{row}}^*$ for the corresponding column and row models. The likelihood ratio L_P^* / L_Q^* measures the goodness of fit of model P vs model Q.

TABLE 8
Likelihood Ratios for Each Pair of Models

Model	Model		
	M	H	MS
M	1.000	0.117	2.11×10^{23}
H		1.000	2.45×10^{21}
MS			1.000

Table 8 shows that model MS is the most successful of the three models in predicting coalition frequencies. Model M is the second best.

Having established the superiority of Model MS over its competitors, we move next to evaluate its success in predicting the frequencies of the observed coalitions. The observed coalitions formed under single-trial agreements are grouped by attraction structures. The five different attraction structures for model MS are shown and exemplified in Table 6. Note that the meanings of i , j , and k are not the same for all five attraction structures. In types I through III, i is the top-ranked player, j has rank 2, and k has the lowest rank. In type IV players i and j are tied for top rank and k has the lowest rank, whereas in type V, i is the top-ranked player whereas j and k are tied for the next rank. Of the two players in type IV who are tied for top rank we shall denote by i the player who is a member of the highest value coalition and by j the other player. Thus $v(ij)_i > v(ik)_i > v(jk)_i$. And of the two players who tie for the lowest rank, the one who is a member of a coalition with the highest value is denoted by j and the other player by k . Thus, for all five types the players are named so that $v(ij)_i > v(ik)_i > v(jk)_i$.

The observed and predicted coalition frequencies by model MS are shown in Table 9 for the five different attraction structures. (The outcomes of the first trial in each sequence were omitted.) The estimated parameter value ($\hat{\epsilon} = 0.22$) was used to calculate the predicted coalition frequencies. Table 9 indicates generally good fit of the MS model; with one exception, the χ^2 values for testing the difference between observed and predicted frequencies are not significant. These results should be interpreted with caution because of the low predicted frequencies in many of the cells and since the same set of three subjects contributed more than once to the data set.

TABLE 9
Observed and Predicted Coalition Frequencies by
Model MS for Five Different Attraction Structures

Type of attraction Structure	Coalition			Total	χ^2
		ij	ik	jk	
I	Observed	19	16	2	0.27
	Predicted	17.43	17.43	2.15	
II	Observed	24	32	4	3.45
	Predicted	20	31.20	8.82	
III	Observed	6	8	6	10.08*
	Predicted	10.40	2.94	6.67	
IV	Observed	7	14	5	0.70
	Predicted	8.66	13.52	3.82	
V	Observed	4	4	0	4.37
	Predicted	5.87	1.66	0.46	

* $p < .05$.

Payoff Division

As the three models differ from one another in the attraction structures they predict, they also differ from one another in specifying who has the bargaining advantage. All three models share the prediction that the payoff to the member of a winning coalition who enjoys a bargaining advantage over his or her partner "will tend to lie toward the former's preferred end of the negotiation range" (Friend et al., 1978, p. 33). "Tend to lie" has been interpreted to mean that the payoff will fall above the mid-point of the negotiation range.

Morrison (1974) proposed to test this prediction by computing the difference between player i 's payoff and the lower bound of the negotiation range divided by the size of the negotiation range:

$$INS_i = (x_{ij} - x_{ij}^-) / (x_{ij}^+ - x_{ij}^-).$$

In the above expression, x_{ij} is player i 's payoff in coalition ij , and x_{ij}^+ and x_{ij}^- are the upper and lower bounds, respectively, of his or her negotiation range. If either $INS_i < 0$ or $INS_i > 1$, then at least one of the three bargaining heuristics is violated. In terms of the numerical index INS_i , the models predict that if player i enjoys a bargaining advantage over his or her coalition partner, then $INS_i > \frac{1}{2}$.

This prediction was tested in games played under single-trial agreements, which satisfy the following three conditions: (a) one of the two members of the winning coalition, say player i , enjoys a bargaining advantage over his or her partner; (b) the negotiation range includes at least two points; and (c) the payoff vector falls in the negotiation range (i.e., $0 \leq INS_i \leq 1$). Table 10 presents the mean and variance of the INS measures for each model. It also shows the number of games that satisfy conditions (a) through (c) above. Table 10 shows that only for model MS is the mean INS significantly larger than $\frac{1}{2}$ ($t = 4.57$; $p < .05$).

TABLE 10
Mean and Variance of the INS Measure by Model

	Model		
	M	H	MS
Mean	.468	.373	.648*
Variance	.099	.071	.082
Frequency	72	86	79

* $p < .05$.

DISCUSSION

Methodological Issues

Before evaluating the predictive success of the three models, several features of our experimental design merit discussion. First, we believe that by using NPER rather than some form of face-to-face bargaining the effects of personality variables, which are not accounted for by the models, have been significantly reduced. It is possible, however, that social norms of fairness and justice are not as strongly manifested under NPER as under face-to-face bargaining. Second, it may be contended that by employing subjects with previous experience in CF games, we might have biased the results in favor of the CF component of the MS model. Although this possibility can only be ruled out by additional experimentation, post-experimental interrogation of the subjects suggests that our subjects understood very well the difference between the two game situations and that carry-over effects, if any, disappeared after short experience with the status game. Third, it may also be argued that the order of communication, which interacts with the status of the players, and the list of messages (Table 5) might have affected the results. Only additional research on games of status using these factors as independent variables can settle this issue. Fourth, it may be recalled that coalition values were chosen on the basis of the outcomes of the preceding trials such that players with a high rank on trial $t - 1$ were rewarded by rendering the CF on trial t more favorable to them. Although this procedure better reflects the primary characteristics of continuing social interactions, it may be argued that it is biased against the hyperopic model because long-term agreements are easier to reach in a stationary environment. This argument, too, should be considered in future research on games of status.

Another methodological issue has to do with our choice of CFs so as to maximize the discriminability among the three models. Because the myopic and hyperopic models are regarded as complementary, not competing hypotheses, it has been argued by one of the reviewers that our attempt to discriminate among the models might have biased the results in favor of the MS model. To check this possibility, note that in class IV of attraction structures in Table 6 the myopic and hyperopic models make the same prediction which differs from the prediction of the mixed-signal model. The coalition frequencies in this class are 7, 14, and 5 for coalitions ij , ik , and jk , respectively (Table 9). Analyzing this class separately yields $\hat{\epsilon}$ values of .45 and .26 for the myopic/hyperopic and the mixed-signal models, respectively. Whether the superiority of the MS model will hold in other situations in which models M and H agree with each other but not with model MS is an open question.

It should also be noted that several aspects of the game environment, which may influence coalition behavior on trial t , are ignored by all three models. (a) The outcomes of trial 1 through $t - 1$ are presented to the subject at the end of trial $t - 1$ and may affect his or her behavior. (b) The history of bargaining from trial 1 through trial $t - 1$ may also provide important information about the bargaining heuristics that underlie the behavior of his or her co-players, their bargaining tac-

tics, their toughness and trustworthiness, and the norms of fairness and justice to which they subscribe. (c) The actual value of t may also be of significance. When the number of trials per sequence is not known, and even if it varies widely from one sequence to another, a player is likely to generate a fuzzy hypothesis about the sequence's duration. He or she may, therefore, behave differently when $t = 1$, believing that the sequence is likely to proceed for several more games, than when $t = 9$, believing that termination of the sequence is imminent.

Model Evaluation

The prediction that the payoff vector lies in the negotiation range was supported by almost two-thirds of the games played under single-trial agreements. There was stronger support for Assumption 1 than for Assumptions 2 or 3, suggesting that subjects adhere more faithfully to simple rules of thumb that concern ranks than to cognitively more demanding rules that concern both ranks and interval positions.

The predictive success of the models cannot be evaluated, however, without knowledge of the size of the negotiation range. The prediction that the payoff vectors lie in the negotiation range is more powerful the smaller the size of the negotiation range is in relation to the CF values. To assess the predictive power of the models, the ratio $|x_{ij}^+ - x_{ij}^-|/v(ij)$ was computed separately for each single-trial agreement game. The resulting mean ratio was 0.253, showing that the negotiation range occupied on the average one-fourth of the corresponding CF value. The proportions of payoff vectors falling in the negotiation range were found previously to equal 0.61. This success rate is significantly higher than the one expected under the null hypothesis that any division of $v(ij)$ is equally likely ($z = 9.91$, $p < .01$).

Despite the statistical significance of the success rate reported immediately above, the failure of all the models to account for slightly more than one-third of the payoff vectors that fell outside of the negotiation range requires explanation. The models tested in this paper assumed that player i who holds no rank-based preference for another player will choose j and k with probabilities in proportion to the probabilities that his or her choice is reciprocated. In a different two-parameter version of the myopic model, Laing and Morrison (1970) assumed that preferences for coalition partners also depend on interval position. If player i is indifferent between two alternative coalition partners on the basis of rank consideration alone, then player i would prefer to form a coalition with j rather than k if his or her expected interval position is larger when forming a coalition with j rather than k . This version states that player i will choose j with probability $1 - \sigma$ and k with probability σ , where $(1 - \varepsilon) > (1 - \sigma) > 0.5$. This inequality assumes that probability of choice is highest when it is based on a rank preference and somewhat lower when based on interval position preference. Data from the original study (Laing & Morrison, 1970) yielded an estimate of $\sigma \simeq \frac{1}{2}$, indicating that interval position had no effect on coalition choices.

It is possible that the definition of interval position requires modification. The interval position, $d_{i,t}$, was originally defined as a linear function assigning equal weights to the point differences $(\delta_{i,t} - \delta_{j,t})$ and $(\delta_{i,t} - \delta_{k,t})$. But why should the two

point differences be weighted equally? Suppose that players A , B , and C are ranked first, second, and third, respectively, at the beginning of trial t . A reasonable assumption is that player A will be more concerned with player B than C because B is more likely to threaten his or her position in the future than C . Similarly, player C is more concerned with his or her point score relative to B than to A , because it is easier for C to overtake or pass in rank player B than A . Player B is probably equally concerned with player C , who may overtake or pass him or her in rank, and player A , whom B wishes to overtake or pass in rank. All of these considerations suggest redefining the interval position by

$$\begin{aligned}d_{i,t} &= \alpha_i(\delta_{i,t} - \delta_{j,t}) + (1 - \alpha_i)(\delta_{i,t} - \delta_{k,t}), \\d_{j,t} &= \alpha_j(\delta_{j,t} - \delta_{k,t}) + (1 - \alpha_j)(\delta_{j,t} - \delta_{i,t}), \\d_{k,t} &= \alpha_k(\delta_{k,t} - \delta_{i,t}) + (1 - \alpha_k)(\delta_{k,t} - \delta_{j,t}).\end{aligned}$$

We would expect that $\alpha_i > \frac{1}{2}$, $\alpha_k < \frac{1}{2}$, and $\alpha_j = \frac{1}{2}$ in our previous example.

With regard to model-dependent predictions, the relative advantage of model MS over its competitors is well established in the present study. Four findings attest to the superiority of model MS: (a) the estimated parameter value $\hat{\epsilon}$, which is lower for model MS than for any of the other two models; (b) the results of the likelihood ratio (Table 8); (c) the relatively good fit of predicted to observed coalition frequencies for different attraction structures (Table 9); and (d) the significant effect that the bargaining advantage according to model MS has on the division of coalition values within the negotiation range (Table 10). Additional studies are required to test the generality of the mixed-signal model, for example, of 3-person CF games of status in which each player's coalitional power, as reflected in the CF values on trial t , is negatively related to his or her ranking at the end of trial $t - 1$.

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