

Navigating congested networks with variable demand: Experimental evidence

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Received 15 March 2006; received in revised form 16 April 2006; accepted 17 April 2006

Available online 24 July 2006

Abstract

We consider traffic networks with a common origin and common destination that are subject to congestion and susceptible to the Braess paradox. Users of such networks choose their routes simultaneously in an attempt to minimize travel cost. A counterintuitive implication of the equilibrium analysis of the users' route choices is that, as demand for the network increases, the addition of a new edge to the network may be beneficial to all the users when the network congestion is relatively low ($n = 10$ in our study), harmful when congestion is moderate ($n = 20$), and have no effect when congestion is high ($n = 40$). Using a within-subjects design with payoff contingent on performance, we test this hypothesis in a network experiment and report evidence that strongly supports it.

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JEL classification: C72; C92; D74

PsycINFO classification: 2340; 2343; 3020

Keywords: Braess paradox; Changes in demand; Experimental study; Traffic networks

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1. Introduction

Networks form the infrastructure for the functioning of modern societies, particularly in the domains of transportation and communication. Trains move on railways, cars travel on highways, and electronic messages are transmitted through telecommunication lines. The design, control, maintenance, and upgrading of such networks require considerable knowledge of fundamental issues in science and engineering. But since networks are designed for and used by human agents, whose rationality is known to be bounded and their cognitive abilities are limited (see, e.g., the voluminous literature on heuristics and biases), equally important for the successful functioning of such networks is the understanding of the *behavioral patterns* that emerge when groups of independent agents have to use these networks repeatedly (see, e.g., Schelling, 1978).

A major question concerns the effects of changes in the network: if the topology of the network or the demand for the network are changed, do users adjust to these changes and if so, can the dynamics of their behavior be accounted for? Answers to this question are important not only to engineers, policy makers, and network managers, but also to social scientists interested in the more general question of how decentralized decision making systems behave in a non-stationary environment as the network users have to adjust to changes in order to achieve whatever goals they have in mind.

We tackle this question experimentally by systematically manipulating changes in demand for network service in the controlled environment of the laboratory. We propose to simulate a simple traffic network and display it on networked computers, recruit subjects who volunteer to participate in a route choice experiment for payoff contingent on performance, repeat the stage game with large groups of subjects for multiple periods to study the effects of outcome information and experience on route choice, systematically change the demand for the network, and study how subjects respond to these changes. We use the Nash equilibrium solution concept as the theoretical benchmark for modeling route choice by the users. Previous experimental studies by Rapoport, Kugler, Dugar, and Gisches (2005a, 2005b, hereafter RKDG) have investigated user reactions to changes in the *topology* of the network. Without changing the network topology, the present study focuses on user reaction to changes in the *demand* imposed on the same network.

The rest of the paper is organized as follows. Section 2 consists of five subsections. The first introduces terminology required to model traffic networks as directed graphs. The second continues with a description and discussion of the network that we implement in the present study. This particular network is susceptible to the Braess paradox (Braess, 1968; hereafter BP) that we describe below in detail. Section 2 continues with two subsections that review the theoretical and experimental literature, respectively, and concludes with a fifth subsection that presents the major hypotheses of the study. Section 3 first describes the experimental design and then the results of the experiment. The results provide strong support for the equilibrium solution and the counter-intuitive behavior that it implies for variable demand. Section 4 concludes the article.

2. Theory

2.1. Terminology

We focus on networks that are modeled by a graph $G = (V, E, O, D)$, where V is a finite set of vertices (nodes), E is a finite set of edges (arcs, links), and O (the origin) and D (the

destination) are two distinct vertices in V . Each edge $e \in E$ has a *tail* $t(e) \in V$ and a *head* $h(e) \in V$; we interpret e as a one-way road segment from $t(e)$ to $h(e)$. A route (path) in the network G is a sequence of the form $v_0, e_1, v_1, e_2, v_2, \dots, v_{g-1}, e_g, v_g$, where $v_0, v_1, \dots, v_g$ are distinct vertices, $v_0 = O, v_g = D$, and $e_1, e_2, \dots, e_g$ are edges satisfying $t(e_i) = v_{i-1}$ and $h(e_i) = v_i$ for $i = 1, 2, \dots, g$.

We consider non-cooperative games with complete information that are played on the network G and have the following ingredients:

- A finite set of players (users) $N = \{1, 2, \dots, n\}$. Transportation science differentiates between the case where n is infinite, so that each user has an infinitely small impact on road congestion, and the case where n is finite. In our study n is finite and commonly known.
- An assignment of costs to edges that may depend on the number of users who traverse these edges. We denote by $c_{ij}(f_{ij})$ the cost to each user of moving along an edge (i, j) with a tail $t(e) = i$ and head $h(e) = j$, if the total number of users of (i, j) is f_{ij} . In the context of traffic networks, $c_{ij}(f_{ij})$ is taken to represent the travel cost for the road segment (i, j) when it is traversed by f_{ij} users.
- In our experimental games, every user has to travel from an origin O to a destination D . The strategy space of each user is the set of routes in G . Route choices are made simultaneously, so that any explicit collusion between users is prohibited. We consider in this study traffic networks with affine cost functions, where for each edge $(i, j) \in E$, $c_{ij}(f_{ij}) = a_{ij}f_{ij} + b_{ij}$ for some non-negative constants a_{ij} and b_{ij} . The affine cost function includes two components: a *fixed* component b_{ij} that is interpreted as the cost of traversing edge (i, j) by a single user, and a *variable* component a_{ij} that when positive accounts for the increase in travel cost due to congestion. We chose affine edge cost functions because they are intuitive and thus most easily explained to the subjects, and because they are supported by empirical evidence (Steinberg & Zangwill, 1983).

2.2. The Braess paradox

Exogenous changes in traffic networks often consist of adding new links or deleting existing links. It seems intuitively obvious that adding new links to a traffic network, and thereby increasing its capacity, should decrease or at least have no effect on the average cost of travel from vertex O to D . Braess showed that if the traffic network is susceptible to congestion then this conclusion may not be true (Braess, 1968; Murchland, 1970). In his seminal paper, Braess presented a very simple model of a congested traffic network with only four vertices and showed that, paradoxically, when a link is added to the network and each driver independently seeks her best possible route from the origin O to the destination D , at the new equilibrium the cost of travel of all drivers may *increase*. Arnott and Small (1994), Cohen and Jeffries (1997), and Vickrey (1969) have discussed other adverse effects of congestion in traffic networks. The BP has attracted considerable attention and instigated much theoretical research in transportation science, engineering, and computer science (Roughgarden, 2005).

To illustrate the BP, consider a finite number n of drivers seeking to travel by car from an origin O to a destination D by means of a network of roads. The complexities of real road networks are approximated by a directed graph. Fig. 1A and B exhibit two examples of weighted directed graphs (note the direction of the arrows) as simple models of road

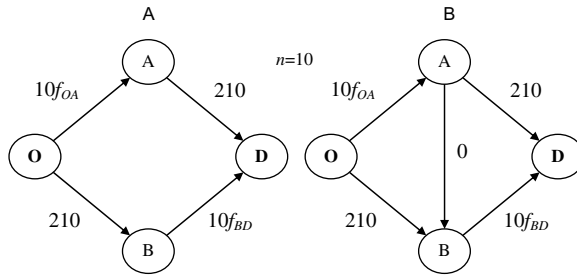


Fig. 1. Basic (A) and augmented (B) networks with four vertices.

networks. The network in Fig. 1B only differs from the one in Fig. 1A by an additional link with a tail A and head B . To sharpen the effects of the BP, we assume that the cost of traversing from A to B is zero: $c_{AB}(f_{AB}) = 0$ for all f_{AB} .¹ The costs associated with all the links are indicated in the graphs (e.g., 210 to traverse the distance from O to B). Note that while the cost of links $(O-A)$ and $(B-D)$ is subject to congestion effects as it depends on the number of users traversing them, links $(A-D)$ and $(O-B)$ have fixed costs that are independent of the number of users choosing to travel on them. Assume that there are $n = 10$ users and that the cost structure is commonly known at the origin O . Equilibrium solutions for the network in Fig. 1A require that five users chooses route $(O-A-D)$ and five choose route $(O-B-D)$. The equilibrium cost for each user is $(10 \times 5) + 210 = 260$, and no user benefits by unilateral deviation to the alternative route.²

Consider next Fig. 1B, which has three (rather than two) routes, namely $(O-A-D)$, $(O-B-D)$, and a new route $(O-A-B-D)$. It is easy to verify that, in equilibrium, all 10 users choose the route $(O-A-B-D)$. The equilibrium cost for each user is now $(10 \times 10) + 0 + (10 \times 10) = 200 < 260$. To establish that this route is an equilibrium, note that any unilateral deviation from route $(O-A-B-D)$ to either route $(O-A-D)$ or route $(O-B-D)$ increases travel cost from 200 to 310. If each user responds to the choices made by the other users, then the flows and associated costs stabilize at the new equilibrium route $(O-A-B-D)$. All 10 users benefit (travel cost is reduced by 23%) from the addition of the new cost-free link $(A-B)$.

The occurrence of the BP depends critically on the choice of the parameter values, namely, n and c_{ij} . To illustrate this dependence, assume the same costs as in Fig. 1A and B, but double the number of users from 10 to 20. It is again easy to verify that, in the equilibrium for the network in Fig. 1A, 10 users choose route $(O-A-D)$ and 10 others route $(O-B-D)$ for an equilibrium cost of 310. In the equilibrium solution for the network in Fig. 1B, all 20 users choose the route $(O-A-B-D)$. The equilibrium cost for each user is now $(20 \times 10) + 0 + (20 \times 10) = 400 > 310$. To verify this counterintuitive result, note that if a user unilaterally deviates from route $(O-A-B-D)$ to either route $(O-A-D)$ or $(O-B-D)$, her cost of travel will increase from 400 to 410. In this exceedingly simple network, if each user responds to the

¹ We have set $c_{AB}(f_{AB}) = 0$ to allow comparison with Experiment 1 of RKGD (2005b). In general, the two parameters in the cost function $c_{ij}(f_{ij}) = a_{ij}f_{ij} + b_{ij}$ can assume any non-negative values (as in Braess, 1968). Moreover, the BP can be realized for any $n \geq 2$. Consider, e.g., a four-node network with $n = 2$, $c_{OA}(f_{OA}) = 10f_{OA} + 6$, $c_{AD}(f_{AD}) = f_{AD} + 30$, $c_{OB}(f_{OB}) = f_{OB} + 32$, $c_{BD}(f_{BD}) = 10f_{BD} + 4$, and $c_{AB}(f_{AB}) = f_{AB} + 3$, that result in equilibrium travel costs of 47 and 55 in Fig. 1A and B, respectively.

² To simplify notation, we denote each edge by its tail and head only, and each route by the vertices along this route, omitting the connecting edges.

choices made by the others, the flows and associated costs also stabilize at the new equilibrium route ($O-A-B-D$). However, all 20 users no longer benefit from the addition of the new link ($A-B$). In fact, their equilibrium travel cost *increases* by about 29%.

There is a flip side to the BP that some may find even more intriguing. In the BP just described above ($n = 20$), the addition of a link to a network causes all users to be worse off in equilibrium. Alternatively, start with the augmented network in Fig. 1B, *delete* the edge ($A-B$) and note that, in equilibrium, all 20 users *benefit* from the degradation of the network.

The latter example with $n = 20$ users is just another illustration that the Nash equilibrium does not, in general, maximize social welfare. Two other well-known examples that dramatically illustrate the Pareto inefficiency of the Nash equilibrium are the n -person Prisoner's Dilemma game (see, e.g., Colman, 1995; Schelling, 1978) and the Centipede game (Aumann, 1992). As noted by RKDG (2005b), the finitely iterated Prisoner's Dilemma game, Centipede game, and network games susceptible to the BP provide complementary perspectives for examining interactive decision situations in which Nash equilibria do not maximize social welfare. They are complementary in the sense that the Prisoner's Dilemma is a strategic form game with dominated strategies, the Centipede game is an extensive-form game with payoffs that vary from one terminal node to another, and the BP is realized for certain parameter values when comparing two networks that differ from each other only by the addition of one or more links. It is the surprising implications of the equilibrium analysis of these games that deepen our understanding of the equilibrium solution, clarify the conflict between individual and group rationality, illustrate the effects of negative externalities (see Schelling, 1960, on the effects of negative externalities in other contexts), and thereby induce our intuition about decentralized decision making to expand.

2.3. Previous theoretical research

Following the discovery of the BP by Braess (1968), researchers have attempted to classify traffic networks in which the addition of a single link could degrade network performance (Frank, 1981; Steinberg & Zangwill, 1983). Dafermos and Nagurney (1984), Smith (1979), and Steinberg and Stone (1988) have discovered new types of “paradoxes”, and Roughgarden (2002) has proved that detecting the BP even in its worst possible manifestation is algorithmically difficult. Roughgarden and Tardos (2002) proved that in networks in which the cost associated with each edge is an affine function of this edge congestion, the flow at the Nash equilibrium has total cost of at most $4/3$ times that of the optimal flow. For further extensions, discussions, and examples of the BP, see Catoni and Pallottino (1991), Fisk (1979), Penchina (1997), Roughgarden (2005), and Pas and Principio (1997).

2.4. Previous experimental research

There are only a few experimental studies of route selection in congested traffic networks. Selten et al. (2004) conducted laboratory experiments of a day-by-day route choice game with two parallel roads but no crossroad that is very similar to Fig. 1A in our study. They reported aggregate road choices that were accounted for quite well by the Nash equilibrium predictions, and large fluctuations around the mean choice frequencies, which did not seem to diminish after a large number of trials. Helbing (2004) reported experiments with more iterations, and further tested additional experimental conditions in an attempt to better understand the reasons for the fairly large fluctuations around the mean choice

frequencies. Iida, Akiyama, and Uchida (1992) conducted similar experiments in Japan with traffic conditions that vary from one day to another. They reported that traffic flow did not converge to equilibrium. All of these experiments have held the network fixed over multiple iterations. None of them has been concerned with changes in behavior due to changes in the topology of the network or in the number of users.

Two other studies of the BP by RKDG (2005a, 2005b) are more directly relevant to our study. We report here only the results of Experiment 1 in RKDG (2005b), which used the same networks in Fig. 1A and B with link cost function parameters $c_{OA}(f_{OA}) = 10f_{OA}$, $c_{BD}(f_{BD}) = 10f_{BD}$, $c_{AD}(f_{AD}) = c_{OB}(f_{OB}) = 210$, and $c_{AB}(f_{AB}) = 0$. Six groups of $n = 18$ subjects each participated in the experiment. Three groups first played Game 1A (Fig. 1A) for 40 identical rounds and then Game 1B (Fig. 1B) for 40 additional rounds (Condition ADD), whereas three other groups played the two games in the reverse order (Condition DELETE). The network and edge costs were displayed on the individual computer screens on each round. After all players independently registered their route choices, each was informed of the number of players choosing each route and her payoff for the trial. Travel costs were subtracted on each period from a fixed endowment that assumed the same value for Games 1A and 1B. The major findings of Experiment 1 are as follows:

- There were no statistical differences between mean route choices in Conditions ADD and DELETE.
- The equilibrium solutions accounted very well for the mean route choices in Game 1A, and the mixed-strategy equilibrium accounted for the variability around the means. In agreement with results reported by Selten et al. (2004) and Helbing (2004), fluctuations of the route frequencies around the mean in Game 1A persisted over all the 40 rounds. However, there was no support for mixed-strategy equilibrium play on the individual level, as most players mixed their route choices but not in the proportions implied by the symmetric equilibrium play.
- On each round, travel costs were subtracted from a fixed endowment of 420 to determine the subject's payoff. Consequently, equilibrium play reduced the payoff per round by 50% from 120 ($420 - 300$) in Game 1A to 60 ($420 - 360$) in Game 1B. Despite this sharp drop in payoff, mean route choices in Game 1B (and the corresponding individual payoffs) in both conditions slowly converged over iterations to the equilibrium strategies. By round 40, all the players in Game 1B in each of the six sessions chose the (deficient) equilibrium route ($O-A-B-D$).

2.5. Hypotheses

We examine in this section the effects of change in demand on equilibrium play in Game 1B and characterize the equilibrium solutions. We first characterize the pure-strategy equilibria. Table 1 presents these equilibria and their associated travel costs for Games 1A (columns 2–4) and 1B (columns 5–8). The results are presented only for even values of n , namely $n = 2, 4, \dots, 42$. When $n = 40$, there are four pure-strategy equilibria in Game 1B. In one of them, 20 agents choose route ($O-A-D$) and 20 choose route ($O-B-D$) for travel cost of 410, and in another 19 choose route ($O-A-D$), 19 choose route ($O-B-D$), and two choose route ($O-A-B-D$) for travel cost of 420. In a third equilibrium, 19 users choose route ($O-A-D$), 20 choose route ($O-B-D$), and 1 chooses route ($O-A-B-D$), whereas in the fourth equilibrium 20 users choose route ($O-A-D$), 19 choose route ($O-B-D$), and

Table 1
Equilibrium analysis of route choice and travel cost in Games 1A and 1B for variable network demand

N	Game 1A			Game 1B				Change in cost
	Route (O–A–D)	Route (O–B–D)	Travel cost	Route (O–A–D)	Route (O–B–D)	Route (O–A–B–D)	Travel cost	
2	1	1	220	0	0	2	40	↓
4	2	2	230	0	0	4	80	↓
6	3	3	240	0	0	6	120	↓
8	4	4	250	0	0	8	160	↓
10	5	5	260	0	0	10	200	↓
12	6	6	270	0	0	12	240	↓
14	7	7	280	0	0	14	280	=
16	8	8	290	0	0	16	320	↑
18	9	9	300	0	0	18	360	↑
20	10	10	310	0	0	20	400	↑
22	11	11	320	1	1	20	420	↑
24	12	12	330	3	3	18	420	↑
26	13	13	340	5	5	16	420	↑
28	14	14	350	7	7	14	420	↑
30	15	15	360	9	9	12	420	↑
32	16	16	370	11	11	10	420	↑
34	17	17	380	13	13	8	420	↑
36	18	18	390	15	15	6	420	↑
38	19	19	400	17	17	4	420	↑
40	20	20	410	19	19	2	420	↑
				19	20	1	410/420	↑
				20	19	1	410/420	↑
				20	20	0	410	=
42	21	21	420	21	21	0	420	=

one chooses route (O–A–B–D). The travel cost associated with these two equilibria is either 410 or 420, depending on the chosen route. The four pure-strategy equilibria are very similar to each other, given the large size of the group (40), and predict that at most two agents choose route (O–A–B–D). If *n* is odd, it is no longer the case that all the *n* agents incur the same cost in equilibrium. However, the *mean* travel cost falls between the costs associated with the adjacent even values of *n*. For example, if *n* = 11, then the mean travel cost in Game 1A is 264.45 (260 < 264.45 < 270).

We next characterize the symmetric mixed-strategy equilibria. In Game 1A, each of the two routes is chosen with equal probability. In Game 1B, each agent chooses either route (O–A–D), (O–B–D), or (O–A–B–D) with probabilities *p*₁, *p*₂ and *p*₃, that are computed from³

³ The three probabilities *p*₁, *p*₂, and *p*₃, and the expected value EV, are solved from the four equations

$$\begin{aligned} \sum \frac{(n-1)!}{j_1!j_2!j_3!} p_1^{j_1} p_2^{j_2} p_3^{j_3} [210 + 10(j_1 + j_3 + 1)] &= EV, \\ \sum \frac{(n-1)!}{j_1!j_2!j_3!} p_1^{j_1} p_2^{j_2} p_3^{j_3} [210 + 10(j_2 + j_3 + 1)] &= EV, \\ \sum \frac{(n-1)!}{j_1!j_2!j_3!} p_1^{j_1} p_2^{j_2} p_3^{j_3} [10(j_1 + j_3 + 1) + 10(j_2 + j_3 + 1)] &= EV, \\ p_1 + p_2 + p_3 &= 1, \end{aligned}$$

where summation is over *j_k* = 0, 1, . . . , *n* − 1 (*k* = 1, 2, 3), *j*₁ + *j*₂ + *j*₃ = *n* − 1, and *p_k* ≥ 0.

$$\begin{aligned}
p_1 = p_2 &= (n - 21)/(n - 1) \quad \text{and} \quad p_3 = (41 - n)/(n - 1), \quad \text{if } 21 \leq n \leq 41, \\
p_1 = p_2 &= 0, \quad p_3 = 1, \quad \text{if } n \leq 21, \\
p_3 &= 0, \quad p_1 = p_2 = 1/2, \quad \text{if } n \geq 41.
\end{aligned}$$

For example, if $n = 26$, then $p_1 = p_2 = 0.2$ and $p_3 = 0.6$.

Although the pure-strategy equilibrium solution for Game 1A may seem obvious, problems of coordination arise because of the multiplicity of equilibria. If n is even, then the number of equilibria is given by $n!/[(n/2)!(n/2)!]$, which can assume a very large value (e.g., 115117520, if $n = 30$). These equilibria are asymmetric with one half of the agents choosing route ($O-A-D$) and the other route ($O-B-D$). They are not Pareto rankable, and there is no focal point (Schelling, 1960) to help agents solving the coordination problem.

The right-hand column of Table 1 shows the effect of adding edge ($A-B$) to the network in Fig. 1A on the equilibrium travel cost. Table 1 can be divided into four regions:

- If $2 \leq n \leq 14$, then all the n agents choose route ($O-A-B-D$) in Game 1B. The addition of the edge ($A-B$) is *beneficial* (or not harmful when $n = 14$).
- If $15 \leq n \leq 21$, then all the n agents choose route ($O-A-B-D$) in Game 1B. The addition of the edge ($A-B$) is *harmful*.
- If $22 \leq n \leq 40$, then only a fraction of the agents choose route ($O-A-B-D$) in Game 1B. This fraction decreases in the value of n . The addition of the edge ($A-B$) is *harmful*. (Note the possible exception if $n = 40$.)
- If $41 \leq n$, then route ($O-A-B-D$) is abandoned. The addition of edge ($A-B$) has *no effect* on the equilibrium travel cost.

Using a within-subject design, the experiment reported in Section 3 below tests these predictions for three selected values of n , namely, $n = 0, 20$, and 40.

3. Experiment

3.1. Method

3.1.1. Subjects

The subjects were 240 students at the Hong Kong University of Science and Technology, who volunteered to participate in a decision making experiment for payoff contingent on performance. The subjects participated in one of two conditions each including three sessions. Each session lasted about 2 h. The mean payoff was HK\$145.⁴

3.1.2. Procedure

All the six sessions were conducted at a large computerized laboratory with 80 terminals. Upon arrival at the laboratory, the subjects were seated with maximum separation between them and handed written instructions. Questions about the procedure and the network game were answered privately by one of the experimenters.

⁴ US\$1.00 = HK7.8. Payoffs were quite attractive. For comparison, an hourly rate for on-campus job is HK\$50.

The instructions⁵ displayed the traffic network in Game 1B, explained the affine cost functions, and illustrated the computation of the travel costs for links with either fixed or variable costs. They also explained the computation of the payoff after subtracting the travel cost from a fixed endowment. The subjects played Game 1B in either four groups of $n = 10$ subjects each, two groups of $n = 20$ each, or a single group of $n = 40$ subjects. We will refer to these as game types (or briefly Games) 1B-10, 1B-20, and 1B-40, respectively. Each of these three game types was iterated 40 times in a fixed-matching design for a total of 120 plays. In Condition I (for “increase”), the three game types were played in the order 1B-10, 1B-20, and 1B-40. In Condition D (for “decrease”), the three game types were played in the reverse order. Because subjects participated in the experiment under a within-subjects design, this design allows testing for the users’ reaction to either increasing or decreasing demand on the same network.

The payoff for each round was computed separately for each subject by subtracting her travel cost from an endowment, $E(n)$, that assumed the same value for all subjects and all rounds for a given n . There were three different endowments, namely, $E(10) = 290$, $E(20) = 490$, and $E(40) = 500$, resulting in the same equilibrium payoff of $290 - 200 = 90$, $490 - 400 = 90$, and $500 - 410 = 90$ per trial in Games 1B-10, 1B-20, and 1B-40, respectively.

To choose one of the routes in Game 1B, the subject had to click with the mouse on the links of this route and then press a “confirm” button. Route choices were self-paced. After all the n group members independently and anonymously registered and subsequently verified their route choices, a new screen was displayed with outcome information about choices and outcomes. The game was played under complete information that included the route chosen by the subject, the number of subjects choosing each of the three routes, and the subject’s own payoff.

After completing the final trial, the subjects were paid their earnings in four randomly chosen trials for each of the three game types 1B-10, 1B-20, and 1B-40. Points were accumulated across the 12 payoff trials and then converted into money according to a commonly known conversion rate. Subjects were paid their earnings individually and dismissed from the laboratory.

3.2. Results

To assess the degree of support for equilibrium play in each game type, the effects of non-equilibrium play have to be considered. In equilibrium, all the players in Games 1B-10 and 1B-20 choose route ($O-A-B-D$). Travel costs in Game 1B-10 increase if one or more players deviate. As shown earlier, if a single player deviates from equilibrium, her travel cost increases from 200 to 310. If all 10 players deviate and divide themselves equally between the two routes ($O-A-D$) and ($O-B-D$), then travel cost increases from 200 to 260 (and the corresponding payoff decreases by a factor of 3 from 90 to 30). The equilibrium is unique and efficient.

The effects of deviation are considerably milder in game type 1B-20. In fact, the BP implies that deviation of all 20 players pays off. If a single player deviates, her travel cost increases from 400 to 410. However, if even half of the players deviate and divide themselves equally between routes ($O-A-D$) and ($O-B-D$), then travel cost of each deviator *decreases*

⁵ The instructions are available on request.

from 400 to 360, and of each of the non-deviators from 400 to 300. Clearly, the temptation to deviate is much stronger in Game 1B-20 than Game 1B-10. The equilibrium is unique and deficient.

Finally, consider game type 1B-40, where the equilibria predict that players divide themselves equally (or almost equally) between routes ($O-A-D$) and ($O-B-D$) and no more than two players choose route ($O-A-B-D$). Due to the difficulties of coordinating a close to even split division in a large group, deviations from such a split can be expected. Under such conditions, deviations to route ($O-A-B-D$) in Game 1B-40 by small subsets of players may, in fact, be rewarded. For example, suppose that 26 players choose route ($O-A-D$) and 14 choose route ($O-B-D$) with respective travel costs of 470 and 350. If four players deviate from route ($O-A-D$) to route ($O-A-B-D$), then their travel cost will *decrease* from 470 to 440. Consequently, the dynamic of play might not favor the equilibrium prediction of no more than two players choosing route ($O-A-B-D$).

There is a consensus that, unless the non-cooperative game is perceived to be trivial, equilibrium play is not reached by introspection. Rather, it is learned with experience. Our analysis above of non-equilibrium play suggests that equilibrium in Game 1B-10 will be reached quickly. The results reported by RKDG (2005b) suggest that equilibrium play in Game 1B-20, which is susceptible to the BP, will be reached rather slowly. RKDG reported that with $n = 18$ convergence to equilibrium in Game 1B-18 required 40 trials. Our analysis further suggests that equilibrium in Game 1B-40 will not be reached. Rather, *on average*, the players will divide themselves equally between routes ($O-A-D$) and ($O-B-D$) with oscillations between these two routes that do not diminish with experience. Because of these oscillations, the choice of route ($O-A-B-D$) by a small percentage of the players will persist.

3.3. Route choice $n = 10$

The results for Game 1B-10 are clear and unambiguous. Altogether, there were 24 groups of 10 members each, 12 in Condition I and 12 in Condition D. During the last 10 trials, all the subjects in both conditions and all the groups converged on route ($O-A-B-D$). Fig. 2A displays the mean number of choices of the three routes for all 40 trials across the 24 groups. As predicted, convergence to equilibrium was reached in 5–7 trials. Out of 240 subjects, 213 chose the equilibrium route on *all* 40 trials. Only 27 subjects deviated by choosing either of the two non-equilibrium routes between 1 and 5 times.

3.4. Route choice $n = 20$

Altogether, there were 12 groups of 20 subjects each who participated in Game 1B-20, six in Condition I and six in Condition D. Using the individual number of choice of route ($O-A-B-D$) in the last 10 trials, we find no ordering effect: the difference between Conditions I and D was not significant ($M_I = 9.87$, $M_D = 9.98$, $F(1, 238) = 1.49$, $p > 0.2$). Further, the six groups within each condition were also not significantly different from each other (one-way ANOVA: $F(5, 114) = 1.15$, $p > 0.3$ in Condition I; $F(5, 114) = 0.84$, $p > 0.5$ in Condition D).

Fig. 2B exhibits the mean number of choices of each of the three routes in Game 1B-20. It shows that convergence to the Pareto inefficient equilibrium was reached in about 15–20 trials. Already on trial 1, about 75% of the subjects were choosing route ($O-A-B-D$). With occasional deviation of no more than two subjects, once the equilibrium was reached all

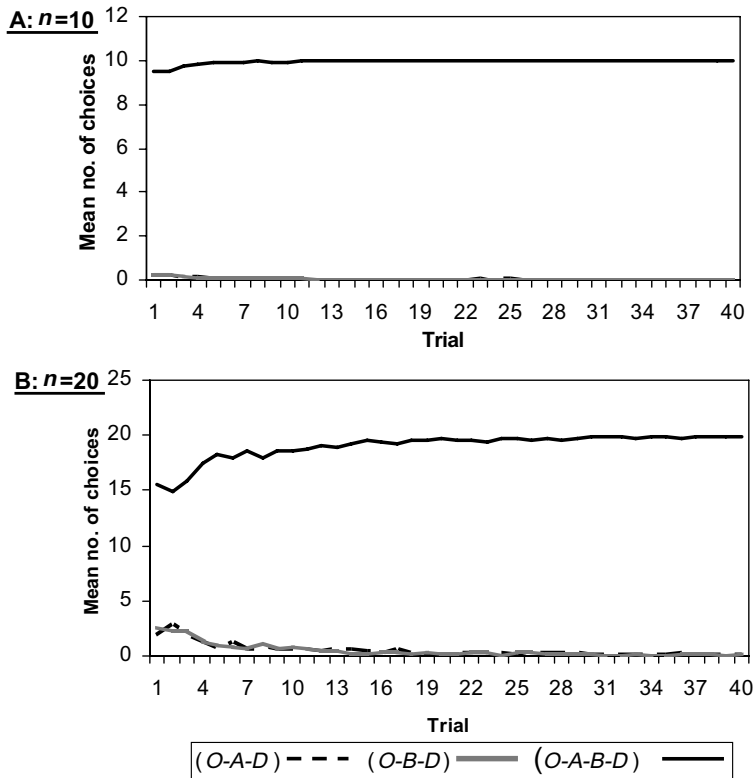


Fig. 2. Mean number of route choices by route and trial for Games 1B-10 (A) and 1B-20 (B).

group members continued choosing route (O-A-B-D) until the end of the session. The results of Game 1B-20 validate the behavioral implications of the BP. Although all group members working in concert could have doubled their payoff from 90 to 180 by deviating from route (O-A-B-D) and dividing themselves equally between the two non-equilibrium routes, the few attempts that might have been made to elicit massive deviation have not been successful. This was true when Game 1B-20 was played after Game 1B-10 in which players converged to choosing route (O-A-B-D), and when it was played after Game 1B-40 where the majority of the subjects chose either route (O-A-D) or (O-B-D) (see below). Therefore, we cannot attribute the results to an inertia effect. Comparison with the results of Experiment 1 of RKDG (2005b) shows that subjects in the present study reached equilibrium twice as quickly. A possible reason for the faster rate of convergence might be due to the difference between the two experimental designs. Whereas subjects in the RKDG study played Game 1B-18 with and without the added link (A-B), subjects in the present study only played the three-route Game 1B-20.

3.5. Route choice $n = 40$

Altogether, there were six groups of 40 subjects each who participated in Game 1B-40, three in Condition I, and three in Condition D. Of the three game types, Game 1B-40 is by

far the most difficult. It does not have a dominant strategy as in Games 1B-10 and 1B-20. Rather, the 40 subjects in Game 1B-40 have to choose one of multiple equilibria in which 19–20 players choose one route and 19–20 another. As shown above, deviation of a small subset of players may, in fact, pay off if equal division between the two equilibrium routes is not achieved.

Fig. 3 displays the frequencies of choice of all three routes in Game 1B-40. The results are presented by group (session), with three groups in Condition I (left panel) and three in Condition D (right panel). All six plots exhibit the same pattern. On the first few trials all three routes were chosen by roughly the same number of subjects. Then, the number of subjects choosing route ($O-A-B-D$) declined sharply but did not go all the way down to zero. On average, 3.68 subjects (out of 40) chose route ($O-A-B-D$) on each of the last 10 trials. Routes

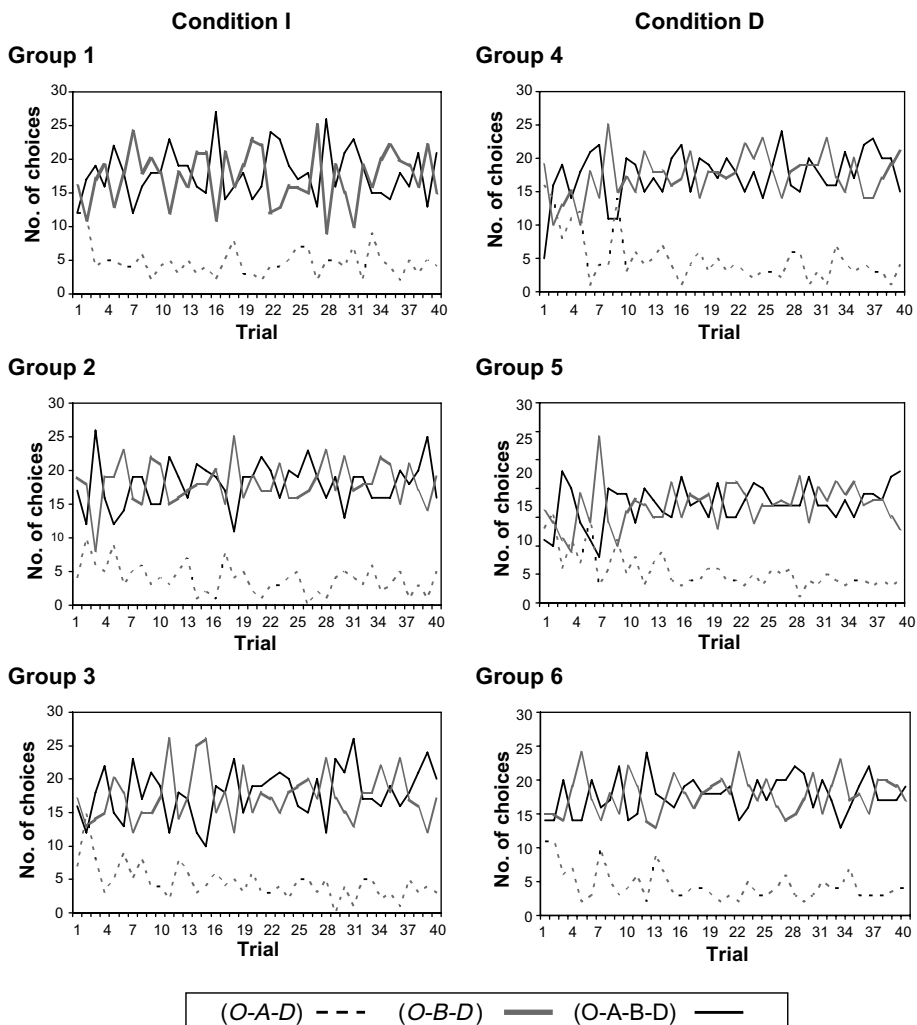


Fig. 3. Number of route choices by route, trial, and group (session) for Game 1B-40.

($O-A-D$) and ($O-B-D$) were chosen by roughly the same number of subjects with oscillations from one trial to another that did not seem to diminish over iterations.

Fig. 4 displays the frequency distributions of choice of route ($O-A-B-D$) in the last 10 trials. The results are shown by group. Statistical analyses failed to show significant differences in individual number of choice of route ($O-A-B-D$) in the last 10 trials between the two conditions ($M_I = 0.93$, $M_D = 0.92$, $F(1, 238) = 0.00$, $p > 0.9$) or between the groups within each condition ($F(2, 117) = 0.55$, $p > 0.5$ in Condition I; $F(2, 117) = 0.07$, $p > 0.9$ in Condition D). Overall, an overwhelming majority of the subjects (67%) did not choose the non-equilibrium route ($O-A-B-D$) even once in the last 10 trials. Rather, only a small minority of the subjects contributed most of these choices.

3.6. Mean payoffs

Route choices translated directly to mean payoffs. Equilibrium mean payoffs, to which we compare the observed results, are computed from Table 1. They assume the values of

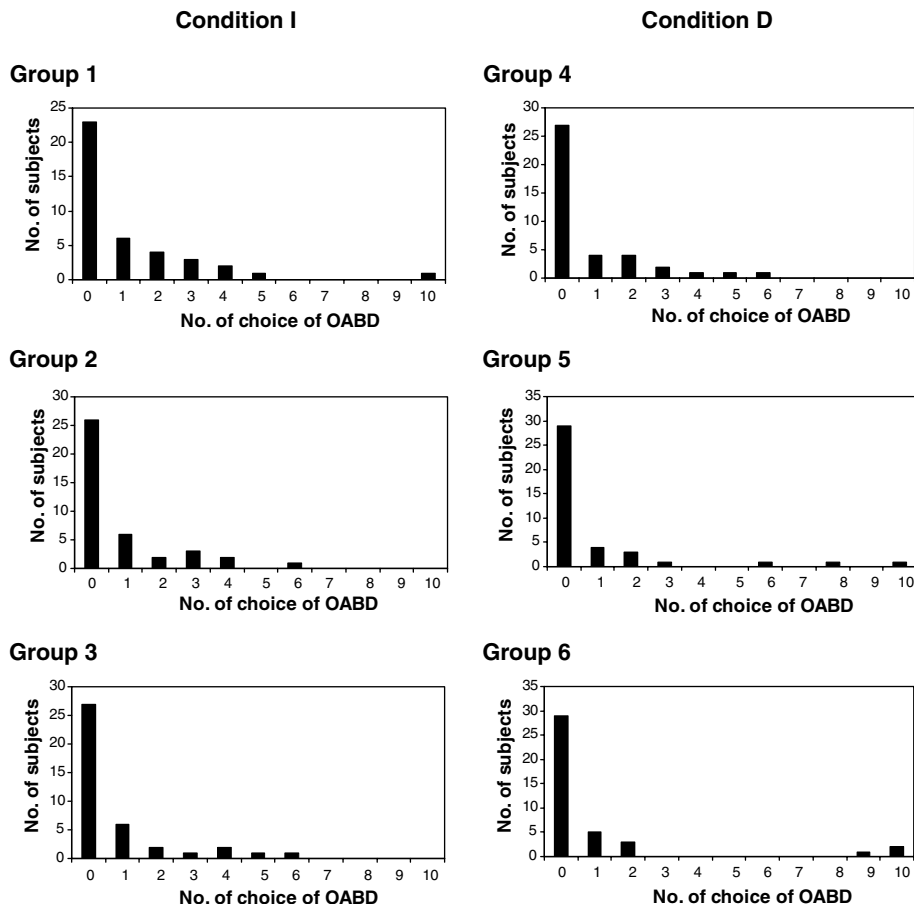


Fig. 4. Number of choices of route ($O-A-B-D$) in the last 10 trials for Game 1B-40 by condition and group.

90 for Games 1B-10 and 1B-20, and either 80 or 90 for Game 1B-40. Fig. 5 displays the predicted (straight lines) and observed mean payoffs for each game type separately by trial. The observed means were computed across 24, 12, and 6 groups in Games 1B-10, 1B-20, and 1B-40, respectively. The mean payoffs in Game 1B-10 converged to equilibrium immediately. The mean payoffs in Game 1B-20 also converged to equilibrium but more slowly. Note that whereas convergence to equilibrium in Game 1B-10 is from below, in Game 1B-20 it is from above. The subjects playing Game 1B-20 could have doubled their payoffs were they to divide themselves equally between routes ($O-A-D$) and ($O-B-D$). In contrast, their mean payoffs decreased over trials. The mean payoffs in Game 1B-40 moved in the direction of the equilibrium payoff, but did not converge. This is because of the 3–5 subjects who persisted, on average, in choosing route ($O-A-B-D$) in the second part of the session. Note the effect that this deviation has on the mean payoffs. For example, if 18,

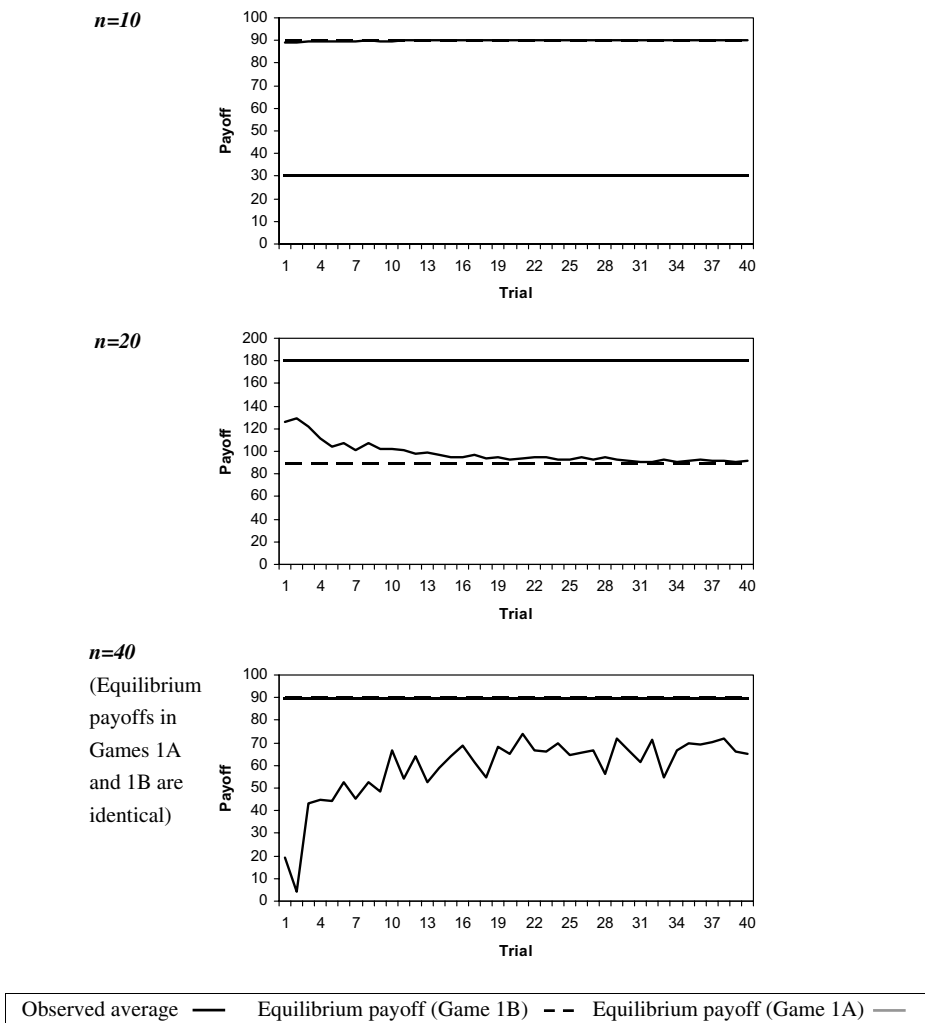


Fig. 5. Mean payoff by game type and trial.

18, and 4 subjects were to choose routes ($O-A-D$), ($O-B-D$), and ($O-A-B-D$), respectively, then the mean payoff would be $500 - 431 = 69$. This value is slightly higher than the observed mean payoff in the second part of the session.

3.7. Switches between routes

Switching routes between trials is a necessary, though not sufficient, condition for convergence to equilibrium. In the present section we analyze the frequency and type of switches between routes. Fig. 6 portrays the frequencies of switches between routes by game type. We distinguish between two types of switches: a switch to/from route ($O-A-B-D$) and a switch between routes ($O-A-D$) and ($O-B-D$). Note the change in scale from one figure to

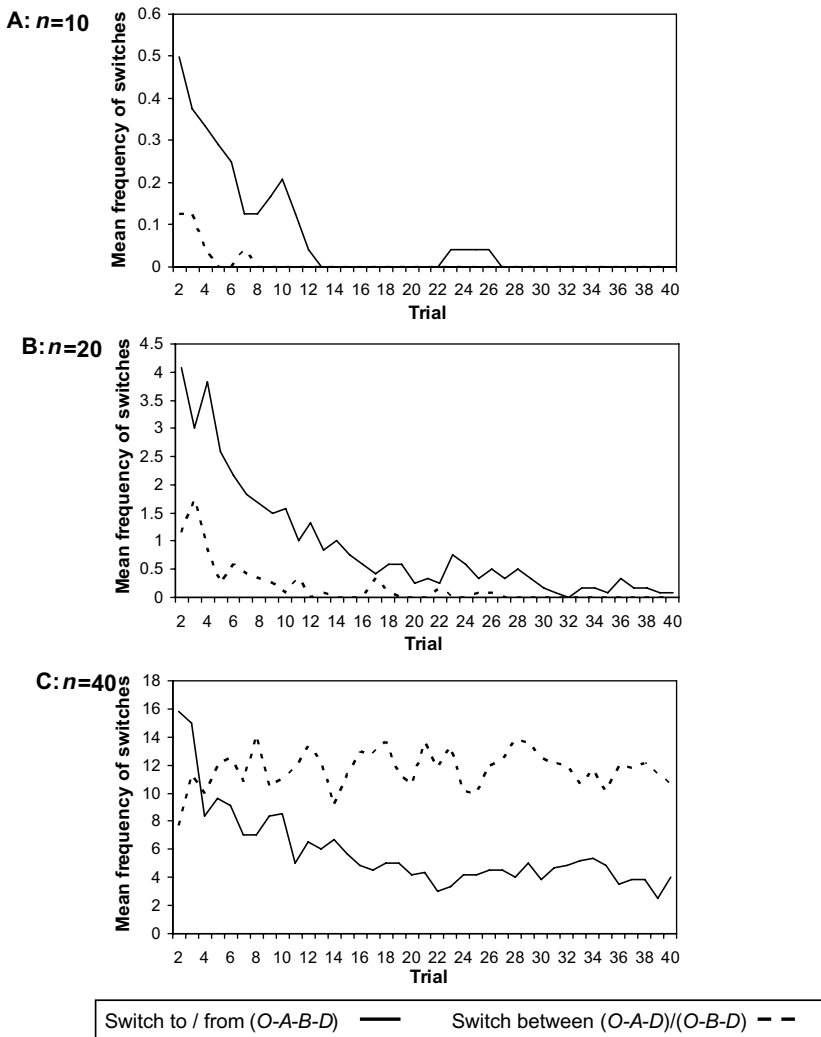


Fig. 6. Mean frequencies of switches by type of switching and trial for each game type.

another. On average, one half of a single subject changed routes to/from route ($O-A-B-D$) in trial 2 in Game 1B-10. The frequency of switches then declined sharply and reached zero in about 14 trials. On average, four out of the 20 subjects in Game 1B-20 switched their route to/from route ($O-A-B-D$) on trial 2. The frequency of switches declined more slowly than in Game 1B-10 and reached zero for the first time after 32 trials. Fig. 6A and B show very few switches between routes ($O-A-D$) and ($O-B-D$) in Games 1B-10 and 1B-20.

On average, 16 of the 40 subjects in Game 1B-40 switched their route on trial 2. The mean number of switches decreased over trials but did not reach zero. On average, 3–5 subjects continued switching their travel route on each trial in the second part of the session. Unlike game types 1B-10 and 1B-20, switches between the two equilibrium routes ($O-A-D$) and ($O-B-D$) for game type 1B-40 stayed more or less constant across the trials after trial 4.

Our results show that switches do not pay off. For each game type separately, we computed the correlation between the subject's payoff for the entire session and her number of switches during the session. All three correlations were negative and statistically significant: $r = -0.89$, -0.14 , and -0.17 for Games 1B-10, 1B-20, and 1B-40, respectively ($p < 0.03$ in each case).

3.8. Type of switches in Game 1B-40

Next, we analyze individual switching patterns in Game 1B-40 in more depth. For each subject, we consider all pairs of route choices in trials t and $t + 1$ ($t = 1, 2, \dots, 39$) and classify them into four categories. Denote the route choice in any trial t by $R(t)$ ($R(t) \in \{O-A-D, O-B-D, O-A-B-D\}$), and define the “realized travel cost” of route R in trial t as the cost of traveling along R given the number of subjects choosing the different routes in trial t . Denote this cost as $C(R, t)$. Then, a route choice in trial $t + 1$ is defined to be:

1. A *fixed response* if $R(t + 1) = R(t)$.
2. A *direct switch* if $R(t + 1) \neq R(t)$ and $C(R(t + 1), t) < C(R(t), t)$.
3. A *contrarian switch* if $R(t + 1) \neq R(t)$ and $C(R(t + 1), t) > C(R(t), t)$.

Any route choice in trial $t + 1$ that does not fall into any of the above categories is classified under the response category “other”. In addition, for each direct or contrarian switch in trial $t + 1$, its “response strength” is defined to be $|C(R(t + 1), t) - C(R(t), t)|$.

For each subject, we counted the total number of each category as well as the mean response strengths for the direct and contrarian types of switches, respectively.⁶ Table 2 lists the major statistics of these dependent variables. The results indicate that the subjects did not switch for more than 56.5% of the time. If they ever made a switch, it was a direct switch for about 55% of the time and a contrarian switch for about 36% of the time. A paired t -test shows that the mean direct response strength is significantly larger than the mean contrarian response strength, $t(228) = 5.21$, $p < 0.0001$. An interpretation of this finding is that subjects are willing to use the more “adventurous” contrarian switch only when the cost difference is not “too large”, while subjects are more easily lured to a direct switch when the cost difference is attractive.

⁶ For those subjects who never use a direct and/or contrarian switch, the corresponding mean response strength is taken as a missing value.

Table 2

Major statistics of individual route choice response measures for Game 1B-40

	Mean	SD	Minimum	Maximum
<i>Response frequencies</i>				
Fixed response	21.93	6.26	7	39
Direct switch	9.64	4.54	0	21
Contrarian switch	6.11	3.88	0	18
Other	1.32	1.23	0	5
<i>Response strengths</i>				
Direct	52.28	15.87	20	120
Contrarian	44.33	17.58	10	120

The correlations between the number of direct and contrarian switches ($r = -0.065$) and between the corresponding response strengths ($r = 0.11$) were not significant. Even more interestingly, there are negative and significant correlations ($r = -0.23, -0.15$) between direct/contrarian response strength and the frequency of the opposite type of switch, while the same type response strength-switch frequency correlations ($r = -0.064, -0.08$) are non-significant. These results suggest that:

1. If a subject prefers direct switches, she will only use a contrarian switch when the cost difference associated with that contrarian switch is small.
2. If a subject prefers contrarian switches, she will only use a direct switch when the cost difference associated with that direct switch is small.
3. Subjects exhibit a wide variety of preference over direct and contrarian switches.
4. A subject's preference over either direct or contrarian switch does not predict whether a large or small cost difference will induce her to use that type of switch.

4. Conclusions

If users of networks that share a common origin and common destination choose their routes simultaneously in a selfish attempt to minimize individual travel cost, then their behavior may give rise in equilibrium to counterintuitive results. One of them is due to Braess, who has shown that if a new edge is added to a simple four-edge network, while keeping the number of users fixed, travel cost of all users may increase in equilibrium. RKDG (2005a, 2005b) have reported evidence from three experiments with different link cost functions or network architecture in strong support of this implication. Pas and Principio (1997) have further demonstrated theoretically that if the topology of the network is held fixed, equilibrium travel cost associated with adding a new edge to the network may first decrease, then increase, and finally stay the same as the number of users progressively increases. The results of the present study support this counterintuitive implication. The effects of increase in demand for the network on travel costs, which only holds for selected cost parameters, is particularly ironic as traffic networks are often expanded in reaction to or anticipation of increase in demand.

Viewed from a wider perspective, our results add support to the claim that as the number of interacting agents grows, the negative consequences of deficient Nash equilibria may

be hard to avoid. Altruism, reciprocity, punishment, and tacit coordination that are quite effective in sustaining cooperation in some non-cooperative games with deficient equilibria when $n = 2$ quickly lose their effectiveness and potency when n increases. Differences in group size may account in part for the conflicting results reported in the literature about the predictive power of the Nash equilibrium solution.

Acknowledgements

Financial support has been provided by a contract F49620-03-1-0377 from the AFOSR/MURI to the University of Arizona. We thank Robin Chark, Vivian Lam, and Francis Fung for their assistance in data collection.

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