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Investment decisions and coordination problems in a market with network externalities: An experimental study

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ABSTRACT

We study decision-making and the associated coordination problems in an experimental setting with network externalities. Subjects decide simultaneously in every round how much to invest out of a fixed endowment; the gain from an investment increases with total investment, so that an investment is profitable iff total investment exceeds a critical mass. The game has multiple, Pareto-ranked equilibria; we find that whether first-round total investment reaches critical mass predicts convergence towards the Pareto optimal full-investment equilibrium. Moreover, first-round investments and equilibrium convergence vary with critical mass and group size in a complex way that is explicable by subtle effects of strategic uncertainty on decision making.

1. Introduction

How do economic agents make decisions on their actions, when the payoff of an action depends endogenously on the collective actions of many agents? This question emerges in many realistic situations. For example, the benefit to a firm in subscribing to a new teleconferencing service depends on how many other firms also subscribe to the service. A new business growth opportunity – such as the Internet in the 1990s or an undeveloped region with tourism potential – is profitable to invest in iff the total amount of investment committed to it is high enough to generate market attention, further development, and profits.

In this paper, we look at a class of these situations that is stylized as investing in the presence of network externalities. A market exhibits (positive direct) network externalities when the gain from an investment increases with the total investment in the market.[2] A common consequence is that an investment incurs a net profit iff the total investment exceeds a critical

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2 Network externalities as discussed here are also called direct network effects. Other types of network effects include indirect network effects, which arise when there is interdependent demand between different products or services in a market. Examples include video consoles and games (Shakar and

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mass. However, whether or not the critical mass can be reached depends on individual agents' investment decisions in the first place: an agent who invests (does not invest) in the market contributes marginally to its success (failure) too, and helps fulfill her own belief, which determines her investment decision in the first place. To put it in another way, agents face coordination problems in this market: everyone knows that the best action is to do what others do – if others invest a lot (so that the critical mass will be reached), it is best for an individual to follow, and vice versa; but when agents make decisions individually, it is difficult to gauge what others' decisions would be. Decision making in the presence of network externalities has therefore been described as a "chicken and egg" problem (Bravo and Siciliani, 2007). The present study is an attempt to shed light on it through experimentation.

We examine people's decisions and the associated coordination problems in a multi-person investment game with a payoff function that exhibits network externalities and critical mass effect. Subjects decide simultaneously in every round how much to invest out of a fixed endowment; the gain from an investment increases with total investment, so that an investment is profitable iff total investment exceeds a critical mass. The game has two pure strategy equilibria: the "full-investment", Pareto optimal equilibrium involves all agents investing all their endowments, while the "no-investment", Pareto inferior equilibrium has no one investing. Our research questions are: under controlled laboratory conditions, (i) How are agents' initial or "first-round" decisions in the investment game – with no previous history of playing this game with the other players – affected by critical mass and group size (i.e. the total number of potential investors)? (ii) Once the game is played repeatedly by the same group of agents, how (if at all) do agents' adjustment of decisions from round to round lead to convergence towards an equilibrium? (iii) How (if at all) do agents' initial decisions predict which equilibrium will eventually be selected? As can be seen, the answer to (iii) potentially relates critical mass and group size with equilibrium selection in this game through the answer to (i). We are interested in the effects of these two independent variables because, first of all, critical mass is an inherent outcome of network externalities and should naturally be considered a possible focus when studying decisions in the presence of network externalities; meanwhile, group size has been shown to have an important impact on behavior in other types of games with multiple, Pareto-ranked equilibria (Van Huyck et al., 1990, 2007), and we surmise that it should also impact decisions and equilibrium outcome in our case.

Indeed, we find that the first-round investment varies in a complex way with critical mass and group size: it decreases with critical mass controlling for group size, but may increase or decrease with group size controlling for the critical mass or the minimum average investment needed to reach critical mass, depending on whether the quantity that is controlled for is high or low. This pattern of effects is then mirrored in the likelihood of convergence towards the Pareto optimal equilibrium. As a result, the first-round investments – which are affected by critical mass and group size – can "make or break" convergence to the Pareto optimal equilibrium because the Pareto optimal equilibrium will be attained iff total initial investment reaches the critical mass.

Our research questions differentiate us from most of the network externalities literature (e.g. Leibenstein, 1950; Rohlfs, 1974; Katz and Shapiro, 1985, 1986, 1992; Arthur, 1989; Brynjolfsson and Kemerer, 1996; Chakravarty, 2003a,b; Park, 2004). Previous studies on network externalities predominantly focus on problems of industrial organization such as the strategic decisions of competing sellers of technological products, rather than the coordination problems among potential adopters. Exceptions include the experimental work of Ruffle et al. (2010) and Devetag (2003). Ruffle et al. study a game with network externalities and critical mass effect, and find that a sufficiently low critical mass is paramount to efficient coordination. Devetag's research is on two versions of a game with critical mass effect, one having a payoff function with an increasing returns component that resembles network externalities. She finds that full information feedback among agents immediately after every round improves coordination. However, the influence of critical mass and group size on subject behavior in such games remains unexplored, and we intend to fill this void with the present study.

Our study also contributes to the broader scope of experimental research on coordination games. Coordination problems among players, when there exist multiple, Pareto-ranked equilibria, have long been major issues in game theory (see Schelling, 1960, 2006; Harsanyi and Selten, 1988), partly because of their widespread occurrence. As Cooper and John (1988) point out, whenever an economic model exhibits strategic complementarities (i.e. any agent's best response is increasing in other agents' actions) it may give rise to multiple equilibria, in which case Cooper and John also show that the equilibria are Pareto ranked under some general conditions. If this happens, the model is prone to coordination failure, meaning that the outcome equilibrium turns out to be Pareto suboptimal (see Cooper, 1999 for a comprehensive treatment; cf. also Vives, 2005; Van Zandt and Vives, 2007). The game we use and many other coordination games in the experimental literature indeed have in common the presence of strategic complementarities and multiple, Pareto ranked equilibria. Our experimental results find analogies as well as discrepancies from previous studies in this field. A major similarity is that the Pareto optimal equilibrium is sometimes, but not always, attained, an observation of many studies that investigate coordination failure, such as Cooper et al. (1990), Van Huyck et al. (1990, 1991, henceforth "VHBB"), Van Huyck et al. (1997, 2007). We also find in our experiment that equilibrium convergence in repeated play is sensitive to actions in the first round and is therefore

Bayus, 2003) and PDA hardware standards and software titles (Nair et al., 2004). Yet another type of network effect is called two-sided or, more generally, multi-sided markets (Rochet and Tirole, 2003; Armstrong, 2006). A two-sided market consists of a firm operating a platform (e.g., a shopping mall) whose profits depend on two or more groups of agents (e.g., retailers and consumers) with interdependent demand for the platform. Both indirect network effects and two-sided markets should be distinguished from direct network effects, which have impact on the utility of one type of investment/consumption category for one group of agents only.

indirectly sensitive to parameters such as group size that influence first-round actions; similar observations emerge in the studies by Van Huyck and colleagues, who employ games with discrete strategy space in which every player's best response is an order statistic of all other players' actions.

The literature on the provision point public goods game (see e.g. Rapoport, 1988; Bagnoli and Lipman, 1989; Isaac et al., 1989; Cadsby and Maynes, 1999; Croson and Marks, 2000) provides more points of comparison. In a provision point public goods game, a public good with a known, fixed payoff is provided to all players iff the aggregate contribution reaches a threshold or "provision point". Typically, provision of public good is a Pareto optimal equilibrium outcome, while no-investment is a Pareto inferior equilibrium. While the provision point is analogous to the critical mass in our case, the Croson and Marks (2000)'s meta-analysis shows that success in public good provision depends positively on the Step Return (SR), which is the aggregate group payoff from the public good divided by the total contribution threshold. This means that public good provision is less likely the higher the threshold, which is analogous to our finding that first-round investment and the likelihood of convergence towards the full-investment equilibrium are non-increasing in the critical mass.

However, the group size effects that we present here (see Sections 3 and 4 for details) are not recorded in the aforementioned studies. Van Huyck et al. (1990, 2007) find that coordination success becomes less likely as group size increases controlling for the payoff function; Croson and Marks (2000) report the same effect controlling for SR. But with the game we study, group size may decrease or increase the likelihood of coordination success. It may appear intuitive to assume that strategic uncertainty – the term VHBB use to describe the decision problem subjects face in coordination games – generally increases when the number of players increases. But whether increased uncertainty necessarily pulls actions away from the Pareto optimal equilibrium (which will be decreased investments in our case) is another question. As we shall argue in our hypothesis formulation and then confirm with data, if coordination failure is highly likely when there is little strategic uncertainty, increasing group size may rather increase first-round investment due to an increase in strategic uncertainty. Such effects on first-round investments are then carried over by our posited dynamics of the game, which is based on a simple reinforcement learning model, to produce similar effects on the likelihood of convergence towards the full-investment equilibrium.

In the following sections, we introduce the game we use in our experiment (Section 2), and suggest, in Section 3, a set of testable behavioral hypotheses that (1) relate critical mass and group size to first-round investments; (2) relate first-round investments to equilibrium convergence; and (3) finally, relate critical mass and group size to equilibrium convergence through (1) and (2). We hypothesize that first-round investments and equilibrium convergence vary with critical mass and group size in a complex way that is explicable by subtle effects of strategic uncertainty on decision making; these hypotheses are motivated by previous literature in coordination games (including the provision point public goods game) and the classic linear public goods game. Moreover, we hypothesize that whether first-round total investment reaches critical mass predicts convergence towards the Pareto optimal full-investment equilibrium due to a reinforcement learning process. Next, in Section 4, we report the experimental results when the game is played with different critical masses and group sizes. Our data show that coordination failure occurs in some, but not all, conditions, and lend support to all our hypotheses. We conclude, in Section 5, with a discussion of the insights that our research provides in terms of decision making in the presence of network externalities as well as in coordination games in general, and finally suggest directions for future research. Our study also consists of an Appendix A in which we present an economic model that produces the payoff function we use for our experiment (Section A.1), a comparison of the similarities and major differences between the game we use and the provision point public goods game (Section A.2), and a sample of the experimental instructions we used (Section A.3).

2. Description and basic properties of the game

Given our research objective of investigating investment decisions and coordination problems in a market with network externalities, we design a multi-person game for experimental implementation with the following properties:

(a) Every player's strategy space is continuous and represents her investment in a market;
(b) Every player's payoff function has a network externalities term that increases with the aggregate investment of all the players;
(c) There is no free riding: a player whose investment is zero cannot benefit from any network externalities generated by the aggregate investment;
(d) A positive investment results in a net profit to a player if the aggregate investment exceeds a critical mass, but it results in a net loss if the aggregate investment falls below the critical mass.

We therefore propose the following simultaneous multi-person game, in which every player has endowment w, any player i may invest any amount, $x_i \in [0, w]$, as her chosen strategy, and the payoff to i given all the players' investments is given by

$$\pi_i = w - x_i + \frac{x_i \sum_{j=1}^{N} x_j}{kw} \quad \text{with} \quad \forall i \quad w \geq x_i \geq 0, \tag{1}$$

where N is the total number of players, k is an exogenous parameter, and w is the endowment of each player (we offer in Section A.1 in Appendix A an economic model of network externalities that leads to the payoff function that is Eq. (1)). It can be easily checked that this payoff function satisfies properties (a)–(d) with a network externalities term, $x_i \sum_{j=1}^{N} x_j / kw$, and a critical mass, kw, that is "non-trivial" when $1 < k < N$. Although Eq. (1) is not the only payoff function that satisfies the required properties, we consider it formally simple enough for convenient experimental implementation; it does capture the features of a market with network externalities in a succinct way.

Methodologically, in using a game with continuous strategy space, we depart from the discrete choice paradigm in many network externalities and coordination game studies. The example of the new business growth opportunity mentioned earlier obviously requires the investors to make decisions on a continuous scale. Another example is a network service that caters to large firms. A firm in the market may decide how many employees should subscribe to the service. This number potentially ranges from zero – meaning that the firm does not use the service at all – to the total number of employees in the firm. The net profit that the firm gains through any subscription level depends on its own subscription level and on the subscription levels of other firms, which can be approximated by all players having a continuous and bounded strategy space (see also the model in Section A.1 in Appendix A). In implementation, such a design allows for more strategic flexibility for players, and also allows us to probe more deeply into the beliefs of agents, as an agent's investment level can be used to gauge her beliefs on the total investment from other agents. The statistical effects that we shall describe – including both main effects and interactions in two-way designs that are crucial evidence for our major hypotheses – are much easier to be detected when subjects can express their beliefs by choosing along a continuous scale with proper incentives, compared with when they are only given a binary choice. Had subjects been asked only to make a binary adoption decision, we would have needed a much larger number of data points to test our hypotheses.

The game represented by Eq. (1) bears some resemblance to the provision point public goods game, in which a public good with a known, fixed payoff is provided to all players iff the aggregate contribution reaches a threshold or "provision point". But, in contrast with the typical provision point public goods game, free riding is not possible in the game we design, and this is a necessary property given our research objective (see property (c) above). To be more precise, $\pi_i > w$ iff (1) $x_i > 0$, and (2) the total investment (including i's own investment) exceeds the critical mass, kw; the total investment must be more than k times the individual endowment to make the investment profitable. A detailed comparison between our game and the provision point public goods game is available in Section A.2 in Appendix A.

To identify the equilibria of the game we use, we first look at the best response of i as a function of other players' investments. Since other players' investments only affect π_i through the sum of all investments (cf. Eq. (1)), i's best response can be written as a function of $S_{-i} = \sum_{j \neq i} x_j$. Hence denote i's best response as the function $x_i{}^*(S_{-i})$, which should, by definition, satisfy:

$$x_i * (S_{-i}) \in arg \max_{x_i \in [0,w]} \left[w - x_i + \frac{x_i(x_i + S_{-i})}{kw} \right].$$

The objective function in the right hand side is quadratic and convex in x_i; thus the maximum is attained only at either $x_i = 0$ or $x_i = w$. To determine which of these investment levels is the best response (i.e. maximizes the objective function) given S_{-i}, it suffices to compare their corresponding payoffs (i.e. values of the objective function), which are w when $x_i = 0$ and $(w + S_{-i})/k$ when $x_i = w$. This leads to the following:

$$x_i * (S_{-i}) = w \quad \text{when} \quad S_{-i} > (k-1)w;$$

$$x_i * (S_{-i}) = 0 \quad \text{when} \quad S_{-i} < (k-1)w;$$

An immediate consequence is that the game is non-trivial only if $1 \leq k \leq N$. If $k < 1$, $(k-1)w < 0 \leq S_{-i}$ whatever the players' investments, so that investing w is a dominant strategy for any player i. If $k > N$, $(k-1)w > (N-1)w \geq S_{-i}$ whatever the players' investment, and investing nothing is a dominant strategy for any player i. If $1 \leq k \leq N$, the game exhibits strategic complementarities locally (Cooper and John, 1988), as $x_i{}^*(S_{-i})$ is non-decreasing and partly strictly increasing in other player's total investment. The rest of this paper is therefore only concerned with cases satisfying $1 \leq k \leq N$. Under this condition, the game has two and only two pure strategy equilibria[3]: (i) a full-investment equilibrium in which $x_i = w$ for all i; (ii) a no-investment equilibrium in which $x_i = 0$ for all i. The full-investment equilibrium is Pareto optimal to the players among all outcomes. This is because, since the payoff function of i is linearly increasing in S_{-i}, the maximum possible π_i is achieved when S_{-i} is at its maximum possible value, which must be $(N-1)w$, while i plays her best response against that S_{-i}, which must then be investing w – and this is exactly the full-investment equilibrium play.

[3] In any pure strategy equilibrium, the investment of any player is either 0 or w, since these are the only possible best responses. Suppose there is a pure strategy equilibrium with some players investing 0 and some investing w. Denote the number of players investing w as r; by our assumption, $N > r > 0$. We must have $rw \leq (k-1)w$, otherwise the players investing 0 will not find it a best response. But then $(r-1)w < (k-1)w$, so that the r players investing w can do better investing 0. This leads to a contradiction. Therefore any pure strategy equilibrium must either have all players investing 0 or all players investing w, implying that there can only be two pure strategy equilibria. That $1 \leq k \leq N$ admits both full-investment and no-investment equilibria is straightforward to prove. Therefore, when $1 \leq k \leq N$, there are two and only two pure strategy equilibria.

If the other players play mixed strategies, the expected payoff of i is:

$$E(\pi_i) = w - x_i + \frac{x_i[x_i + E(S_{-i})]}{kw}.$$

Thus, if i is risk neutral, her best response to others' strategies (which would maximize her expected payoff) can only be 0 or w. This implies that, if all players are risk neutral, any mixed strategy equilibrium must only involve mixing between investing 0 and w, and every player's expected payoff in that equilibrium must be w (the payoff when investing 0), which is the same as in the no-investment equilibrium and is Pareto inferior to the full-investment equilibrium. It is straightforward to derive that the only symmetric mixed strategy equilibrium has each player investing w with probability $q = (k-1)/(N-1)$ and investing 0 with probability $1-q$, giving each player an expected payoff of w. Given the focal nature of the pure strategies, the disincentives to be unpredictable in this task, and the fact that the expected payoff of the mixed strategy equilibrium can be guaranteed by the no-investment strategy ($x_i = 0$), we do not expect experimental subjects to attain any mixed strategy equilibrium.

More generally, if players are risk neutral and always play the best-response strategy for the stage game based on their probabilistic belief of what other players may play, then we shall only observe investments of either 0 or w in our experiment. However, we do not expect this to be the case for two reasons. First, a player may be risk averse and invest an amount that is less than w even if she believes that the expected total investment from other players exceeds kw. Second, if, in any round before the last, a player believes that the expected total investment from other players is less than kw, she may still invest a positive amount to "cue" other players to coordinate in reaching an efficient outcome.[4] Hence we expect to observe a significant proportion of players investing amounts other than 0 or w (at least before an equilibrium is reached in a finitely repeated game).

Our experimental setting is a fixed-matching, finitely repeated game with the stage game represented by Eq. (1). The feedback to any player after any round, apart from her own investment and payoff, consists only of the total investment. Our objectives are to observe first-round investments, dynamics, and equilibrium convergence, in conditions with different critical mass and group size.

3. Behavioral hypotheses

In this section, we present a number of behavioral hypotheses regarding how subjects play the game in our experiment. We shall then test the hypotheses with experimental data in the next section.

3.1. First-round investment

While equilibrium selection (as a convergence result) is an important issue in this study, we are also interested in the first-round investments despite their "noisy" nature. The reasons are twofold. First, it is not difficult to imagine real-life situations in which the game is virtually one-shot or lasts few repeated interactions, so that agents' overall gains or losses depend crucially on how much to invest in a new market with little information regarding how other agents might decide. Second, given our hypotheses of the dynamics of this game (to be presented next), first-round investments are crucial in determining whether the full-investment or no-investment equilibrium is reached.

Our aim is to investigate how x_{i1}, the first-round investment of any subject, i, might vary with critical mass and group size as represented by the parameters k and N (we shall keep the endowment w constant across all conditions). We first consider how x_{i1} might change with k holding group size and endowment constant. We take our cue from research in linear public goods games, with which it has been found that the Marginal Per-Capita Return (MPCR, the marginal increase in public good payoff incurred by a unit increase in an individual's contribution) positively influences contribution behavior (Isaac et al., 1984). Croson and Marks (2000) extend the concept behind MPCR to the provision point public goods game with the notion of SR (see Section 1 for definition). They similarly find that SR has a major impact on the success rate of public good provision. There is no straightforward analogy in our game to either MPCR or SR. However, the spirit behind those measures is the marginal increase in public good payoff to a subject per unit increase in the subject's private good contribution. Translated to our game, the term in π_i that most resembles a "public good payoff" (despite the important difference that there is no free riding) is $\left[x_i \sum_j x_j\right]/kw$. And we have:

$$\frac{\partial}{\partial x_i}\left(\frac{x_i \sum_j x_j}{kw}\right) = \frac{\partial}{\partial x_i}\frac{x_i(x_i + S_{-i})}{kw} = \frac{2x_i + S_{-i}}{kw},$$

which is decreasing in k holding the investments and endowment constant. That is, ceteris paribus, the marginal increase in the gain of i's investment due to a small increase in i's investment itself is a decreasing function of k. Extending the

[4] This is different from signaling as usually discussed with repeated game equilibrium, as there is no issue of adverse selection here.

insights from linear and provision point public goods game, we then hypothesize that first-round investment decreases with k holding the group size and endowment constant, or:

H_1. x_{i1} increases with k controlling for N and w.

The variation of x_{i1} with N is more complicated. First, we need to clarify how we should formulate the related hypotheses: what is to be held constant as N changes? Here we follow Isaac et al. (1984)'s study on linear public goods game, in which they formulate hypotheses for group size effect controlling for MRCP and the "group production technology", respectively. To be more specific, they study a game with payoff function Θ_i defined as follows:

$$\Theta_i = w - x_i + \frac{G\left(\sum_j x_j\right)}{N},$$

where x_i is the contribution of i and $G(\cdot)$ is a function of the total contribution only. Controlling for MRCP means controlling for the function $G'(\cdot)/N$, while controlling for "group production technology" means controlling for the function $G(\cdot)$. Following earlier discussion, controlling for MRCP is analogous to controlling for k in our settings (the endowment being made constant in all conditions); this also has the intuitive appeal of controlling for the form of the payoff function. Next, for "group production technology", we first write our payoff function in a form that is more readily comparable with Θ_i, i.e.:

$$\pi_i = w - x_i + \frac{H\left(x_i,\ \sum_{j\neq i} x_j\right)}{N}, \quad \text{where } H\left(x_i,\ \sum_{j\neq i} x_j\right) = \frac{N x_i\left(x_i + \sum_{j\neq i} x_j\right)}{kw}.$$

This suggests that controlling for $G(\cdot)$ in Isaac et al.'s case is analogous to controlling for $H(\cdot,\ \cdot)$ in ours, which means controlling for N/kw. This is equivalent to controlling for kw/N, which also has an intuitive appeal, as it is the minimum average investment needed to reach critical mass. In our experiment, as the endowment is made constant in all conditions, controlling for kw/N is equivalent to controlling for k/N. Therefore, to sum up, we shall formulate hypotheses for group size effect controlling for k (effectively the critical mass) and k/N (effectively the minimum average investment needed to reach critical mass), respectively.

We next derive our hypotheses in detail. Van Huyck et al. (1990, 2007) (see also Crawford, 1995; Broseta, 2000) suggest that, in general, strategic uncertainty increases as group size increases—an intuition that is transferrable to our case. Denoting each condition of the game (controlling for w) by the order pair (N, k), we thus proceed with the following reasoning: (a) Assume H_1 is correct. Suppose initially the parameters of the game are (N_1, k_1). If k_1 is small compared with N_1 (or, equivalently, k_1/N_1 is small compared with unity), x_{i1} must be relatively large, so that first-round investments are nearer the full-investment equilibrium than the no-investment equilibrium; (b) the parameters are then changed to (N_2, k_2), where $N_2 > N_1$, while either $k_1 = k_2$ (i.e. controlling for k) or, in another hypothesis formulation, $k_1/N_1 = k_2/N_2$ (i.e. controlling for k/N). In either case, we hypothesize that the change induces higher strategic uncertainty among players, so that first-round investments are "pulled away" from the full-investment equilibrium, thus resulting in a decrease in first-round investments. To sum up:

H_{2A}. Suppose the parameters of the game change from (N_1, k_1) to (N_2, k_2), where $N_2 > N_1$, while w is kept constant. Then:

(a) If $k_1 = k_2$ and k_1 is small compared with N_1, x_{i1} decreases as a result of the change;
(b) If $k_1/N_1 = k_2/N_2$ and k_1/N_1 is small compared with unity, x_{i1} decreases as a result of the change.

However, if k_1 is close to N_1 (or k_1/N_1 is close to unity – recall that we are only concerned with the non-trivial case of $1 \le k \le N$) a similar line of reasoning leads to an opposite conclusion: (a) assume H_1 is correct. Suppose initially the parameters of the game are (N_1, k_1). If k_1 is close to N_1 (or, equivalently, k_1/N_1 is close to unity), x_{i1} must be relatively small, so that first-round investments are nearer the no-investment equilibrium than the full-investment equilibrium; (b) the parameters are then changed to (N_2, k_2), where $N_2 > N_1$, while either $k_1 = k_2$ (i.e. controlling for k) or, in another hypothesis formulation, $k_1/N_1 = k_2/N_2$ (i.e. controlling for k/N). In either case, we hypothesize that the change induces higher strategic uncertainty among players, so that first-round investments are "pulled away" from the no-investment equilibrium, thus resulting in an increase in first-round investments. To sum up:

H_{2B}. Suppose the parameters of the game change from (N_1, k_1) to (N_2, k_2), where $N_2 > N_1$, while w is kept constant. Then:

(a) If $k_1 = k_2$ and k_1 is close to N_1, x_{i1} increases as a result of the change;
(b) If $k_1/N_1 = k_2/N_2$ and k_1/N_1 is close to unity, x_{i1} increases as a result of the change.

3.2. Dynamics

We now formulate hypotheses for the dynamics of the game after round 1. Our purpose is to relate first-round investments to equilibrium convergence, and as such, we look for hypotheses that require parsimonious parametrization and assumptions. We thus assume that subjects use the following simple reinforcement learning rule to adjust their investments from round to round (see Erev and Rapoport, 1998 for a more elaborate use of reinforcement learning models in coordination game experiments): if a subject invests a positive amount in round $t-1$ and makes a (net) profit or breaks even in that round, she increases her investment in round t unless her investment in $t-1$ is already w, in which case she invests w again in t. But if she invests and incurs a (net) loss in $t-1$, she decreases her investment in t. If she does not invest in $t-1$, she considers what would have happened had she invested a very small but positive amount in that round; if she would have incurred a loss, she invests zero again in t; otherwise, she increases her investment to a positive level in t. Essentially, what we assume is that a subject will "increase her bet" in the next round if she "wins" in the current round, and vice versa. Note that an investment in t-1 makes a profit/breaks even iff $S_{t-1} \geq kw$. Thus, formally, our hypotheses are that, for any round t ($T \geq t > 1$):

H₃. If $S_{t-1} \geq kw$, then

(a) if $x_{it-1} = w$, then $x_{it} = x_{it-1} = w$;
(b) if $x_{it-1} = w$, then $x_{it} > x_{it-1}$.

H₄. If $S_{t-1} \geq kw$, then

(a) if $x_{it-1} = 0$, then $x_{it} = x_{it-1} = 0$;
(b) if $x_{it-1} > 0$, then $x_{it} < x_{it-1}$.

If hypotheses H_3 and H_4 are valid, any subject's investment will be strictly increasing (unless it has reached w) from round to round in the finitely repeated game if the sum of the investments in round 1 reaches the critical mass. Conversely, any subject's investment will be strictly decreasing (unless it has reached 0) from round to round in the finitely repeated game if the sum of the investments in round 1 fails to reach the critical mass. Since any investment is bounded by 0 and w, both monotonic sequences must approach some finite limits as the number of rounds becomes large; two natural candidates for these limits are the equilibrium investments. Hence, we also set out the following hypothesis to be tested:

H₅. The game converges towards the full-investment equilibrium if $S_1 \geq kw$; it converges towards the no-investment equilibrium if $S_1 < kw$.

We now propose hypotheses regarding how the likelihood of convergence towards full-investment equilibrium varies with k and N. This is obviously an important variable as it indicates the likelihood of coordination success. To proceed, we need to relate the first-round investment hypotheses with H_5. Consider, for example, the case when there is an increase in k controlling for N and w from one experimental condition to another. Suppose that, for each condition, one group of subjects is chosen at random from the same, large population, to play the game. Consider how the probabilistic distribution of x_{i1} and S_1 might then differ between the groups playing the two conditions. By H_1, we predict a shift in the distribution of x_{i1} and hence S_1 towards lower values as k increases. That is, the probabilistic distribution of x_{i1} and S_1 in the condition with lower k first-order stochastically dominates that in the other condition. Nevertheless, if the distribution starts out to lie largely above kw, it may still lie largely above (the new) kw after the increase. Conservatively speaking, the likelihood of convergence towards the full-investment equilibrium–which, by H_5, is equal to the probability that $S_1 \geq kw$ – may be the same in both conditions. But this likelihood at least cannot be higher after the increase in k, because of the predicted shift in the distribution of S_1 and the fact that kw has increased. Thus we hypothesize that:

H₆. The likelihood of convergence towards the full-investment equilibrium is non-increasing in k controlling for N and w.

Next consider a change of parameters pertaining to H_{2B}(a) i.e. an increase in N controlling for k and w, where k is close to the initial value of N. The hypothesis predicts that the distribution of x_{i1} will shift towards higher values. This, together with the fact that N has increased while kw is kept constant, leads to us to hypothesize that:

H₇. Suppose the group size changes from N_1 to N_2, where $N_2 > N_1$, while k and w are kept constant, and k is close to N_1. Then the likelihood of convergence towards the full-investment equilibrium is non-decreasing as a result of the change.

However, we cannot make a corresponding hypothesis regarding the change of parameters pertaining to H_{2A}(a), because, while the distribution of x_{i1} will shift towards lower values with an increase in N in that context, the fact that N itself has increased makes it ambiguous whether the distribution of S_1 will shift towards any definite direction. Similarly, for the changes of parameters pertaining to H_{2A}(b) and H_{2B}(b), since k increases together with N in the context of both hypotheses (so that k/N is kept constant), it is ambiguous as to whether a shift in the distribution of x_{i1} as predicted by these hypotheses will lead to a higher or lower likelihood of $S_1 \geq kw$.

4. Experiment

4.1. Method

4.1.1. Subjects

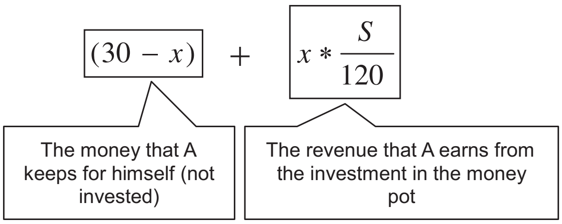
Two hundred and forty undergraduate subjects at a university in Hong Kong participated in the experiment. All the subjects volunteered to take part in a decision-making experiment with payoff contingent on performance.

4.1.2. Design and procedure

We used a between-subject design with five conditions. All the games comprised 20 rounds ($T = 20$) with fixed matching. Each subject had an endowment of $w = 30$ francs (the experimental currency) at the beginning of each round. The conditions differed in N, the total number of players in a game, and k, a measure of the critical mass. By denoting each condition by the ordered pair (N,k), the experimental conditions were (5,2), (5,4), (10,2), (10,4), and (10,8), respectively. Note that we have not included the condition (5,8) to form a complete 2×3 design because (5,8) is not a "meaningful" condition: the critical mass, $8w$, is larger than the maximum possible total investment, $5w$. To keep the incentive constant for all subjects, we varied the exchange rate between francs and the subjects' real currency across conditions so that the maximum possible real currency payoff per round per subject ($= Nw/k$), which was achievable at the full-investment equilibrium, was the same in all conditions and equal to HK$75 (US$1 = HK$7.8).

There were six groups in each condition. Subjects were randomly assigned into conditions and groups, and they were seated at maximal distances apart in front of computer terminals through which the games were conducted. At the beginning of each session, subjects read the instructions about the "money pot investment game" that they were going to play. They were also informed about their endowments, w, in francs, their condition, N, the critical mass, kw, and the exchange rate between the experimental and real currency. The payoff function (1) was presented in the instructions in the following way (this is for the case $k = 4$):

> For every 1 franc that a player invests in the money pot, the money pot will return an amount that is equal to the sum of investments from all the players (including him) divided by 120. That is, if a player has invested x francs in the money pot, and the sum of investments from all the players (including himself) is S francs, then the money pot will return S/120 francs for every 1 franc of his investment, or a total of x*(S/120) francs.
> *Payoff in a Round*. Suppose Player A invests x francs in the money pot and therefore keeps $(30 - x)$ francs for himself in a round. Suppose the sum of investments from all the players in that round, including Player A's own investment, is S francs. Then:
> Player A's payoff in this round [in francs] is:



The full instructions for condition (5,2) are in Section A.3 in Appendix A. It was explicitly stated in the instructions that an investment is profitable iff total investment exceeds the critical mass, kw. As the game proceeded, after each round, each subject was informed about the total investment in that round, in addition to her own payoff and investment. After all 20 rounds were played, two rounds were chosen at random. Each subject was paid her average payoff in real currency in the chosen rounds plus a show-up fee of HK$10 and then dismissed.

4.2. Basic results and analysis

Fig. 1A and B shows the mean investment normalized by w, that is, St/Nw, over all 20 rounds for all groups in all conditions. For ease of exposition, each sub-figure represents data from three groups. The game clearly converged towards the full-investment equilibrium in conditions (5,2), (10,2), and (10,4); we call these conditions the "w-equilibrium conditions". The game predominantly converged towards the no-investment equilibrium in conditions (5,4) and (10,8); we call these the "0-equilibrium conditions". Note that one group out of the six in the (5,4) condition did succeed in attaining the full-investment equilibrium.[5] In general, convergence towards the no-investment equilibrium was "noisier" than towards

[5] A closer inspection of individual data reveals that one player in this group continued to invest at least 2/3 of her endowment in the first three rounds despite other players sharply decreasing their investments at the same time. His/her insistent signaling successfully led to two other players' high-investment responses in round 4 and the game moved gradually towards the full-investment equilibrium afterwards.

Table 1
Mean data (standard deviation) and percentage convergence to the full-investment equilibrium, by condition.

	$N=5$		
	$k=2$	$k=4$	–
First-round investment, normalized wrt w	.57 (.38)	.35 (.40)	–
S_t/Nw averaged over the last five rounds	1.00 (.01)	.20 (.40)	–
Percentage convergence to the full-investment equilibrium	100%	16.7%	–
	$N=10$		
	$k=2$	$k=4$	$k=8$
First-round investment, normalized wrt w	.48 (.37)	.46 (.34)	.45 (.43)
S_t/Nw averaged over the last five rounds	.99 (.01)	.99 (.01)	.04 (.07)
Percentage convergence to the full-investment equilibrium	100%	100%	0%

Note: The standard deviations for first-round investments are calculated with individual players as the unit of observation; all other standard deviations are calculated with groups as the unit of observation.

the full-investment equilibrium, possibly because of players' tendency to cue other players to "resist" the Pareto inferior equilibrium.

Table 1 shows the means of the last five rounds' average investments (normalized with respect to Nw). A non-parametric sign test shows that S_t averaged over the last five rounds is not significantly different from Nw for each w-equilibrium condition and not significantly different from 0 for each 0-equilibrium condition, with $p > .1$ in both cases. This supports the observation about equilibria stated in the last paragraph. For each group, we define that it has converged towards the full-investment equilibrium iff S_t averaged over the last five rounds is at least $0.95Nw$. Table 1 lists the percentage of groups

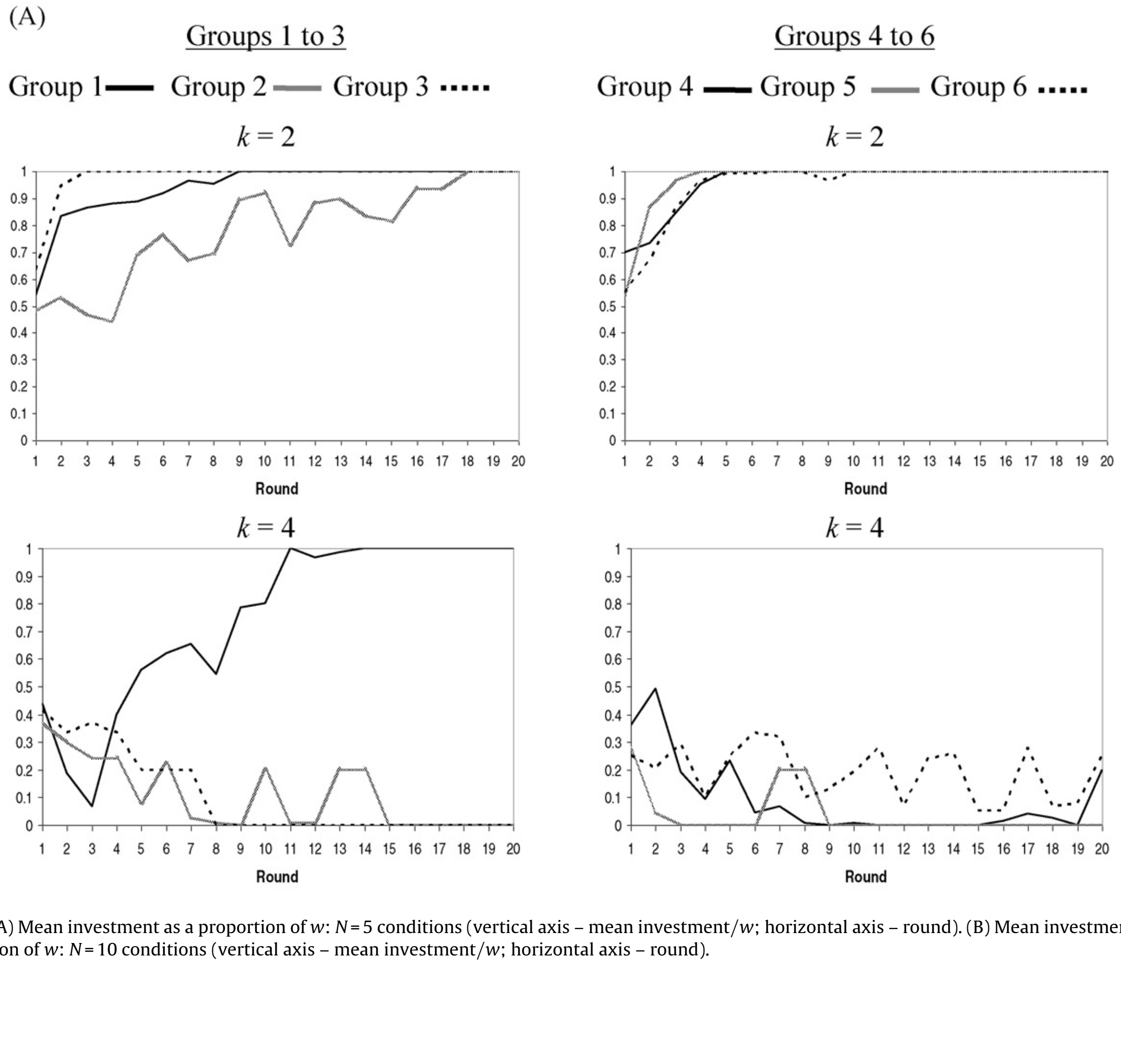


Fig. 1. (A) Mean investment as a proportion of w: $N=5$ conditions (vertical axis – mean investment/w; horizontal axis – round). (B) Mean investment as a proportion of w: $N=10$ conditions (vertical axis – mean investment/w; horizontal axis – round).

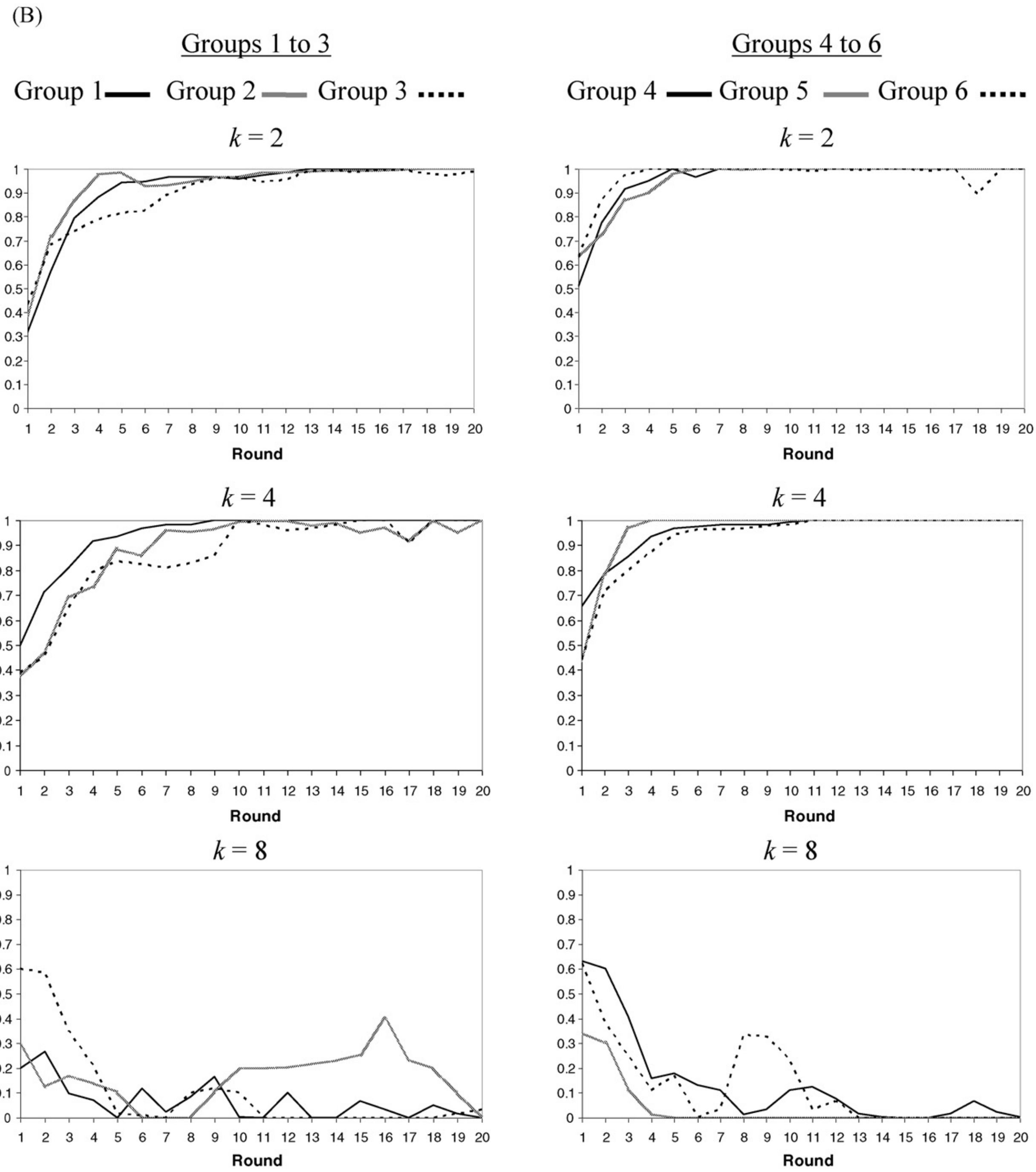


Fig. 1. (*Continued*).

which have converged to the full-investment equilibrium in each condition; an inspection of this variable across conditions again reflects our observations in the last paragraph.

Fig. 2 shows the distributions of the first-round investments under different conditions. Kolmogorov–Smirnov tests show that the distributions for (5,2), (5,4), and (10,8) are significantly different from the uniform distribution at $p < .05$, while those for (10,2), (10,4) is significantly different from the uniform distribution at $p \approx .1$. Overall, the results indicate that first-round investments are not random decisions. The distributions between the w-equilibrium conditions and the 0-equilibrium conditions are also dramatically different. In every condition, at least half of the players invested neither 0 nor w in round 1. This lends support to our expectation that, before an equilibrium is reached, a significant number of players does not play as if they responded to their probabilistic belief in what other players may play in a risk-neutral manner.

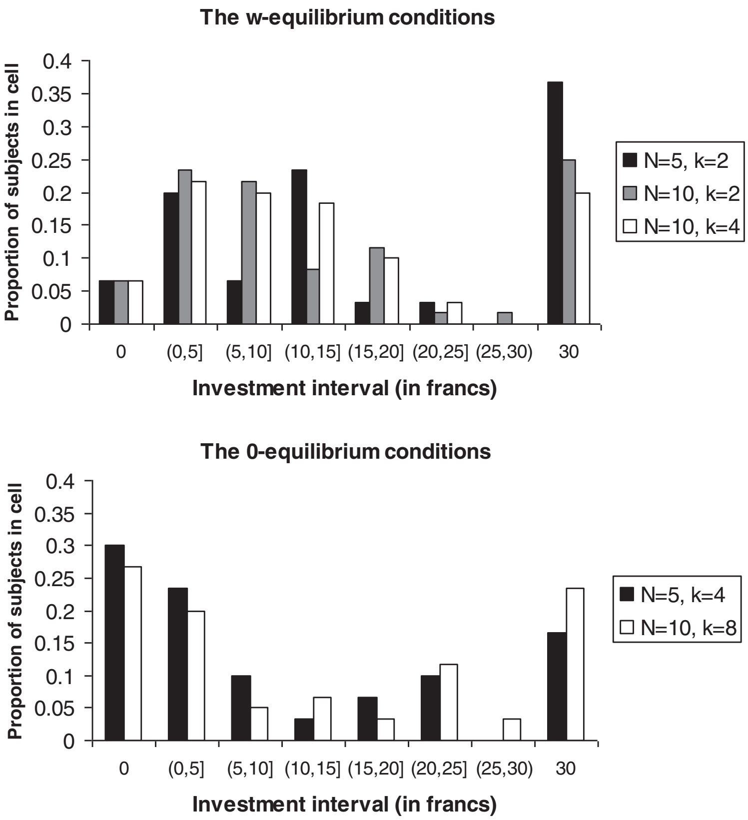


Fig. 2. Histogram of first-round investment by condition.

Finally, it is observed from our data that, in each of the games, individual investments predominantly stabilized around 0 or w as the game proceeded, but they did not fluctuate over these two extreme values. This indicates that the risk-neutral mixed strategy equilibrium was indeed not attained.

4.3. Testing the hypotheses

4.3.1. First-round investment

The means of the first-round investments are shown in Table 1. Hypothesis H_1 is supported for $N=5$ with statistical significance ($t(1)=2.16$, $p<.05$); this indicates also that first-round investments are indeed sensitive to the critical mass. However, the hypothesis is only directionally supported for $N=10$ without statistical significance ($F(2,177)=.13$, $p>.8$).[6] To confirm these conclusions, we leave out condition (10,8) and group the conditions (5,2), (5,4), (10,2), (10,4) into a 2 ($N=5,10$) $\times$ 2 ($k=2,4$) design. ANOVA then reveals a significant main effect in k ($F(1,176)=4.16$, $p<.05$) but also a marginally significant interaction ($F(1,176)=2.99$, $p=.086$), suggesting again that H_1 is only supported statistically at $N=5$. We thus conclude that the data provide partial support for H_1 statistically and are consistent with H_1 directionally.

Another interpretation of the aforementioned interaction is that it provides support for hypotheses H_{2A}(a) and H_{2B}(a). Indeed, the data are consistent with both hypotheses directionally; moreover, ANOVA does not reveal a significant main effect in N at $p<.1$, which is also consistent with the two hypotheses taken together. However, none of the pairwise comparisons testing simple effects in N at $k=2$ and $k=4$ yields significant results at $p<.1$. Next, to test hypotheses H_{2A}(b) and H_{2B}(b), we leave out condition (10,2) and group the conditions (5,2), (5,4), (10,4), (10,8) into a 2 ($N=5,10$) X 2 ($k/N=2/5,4/5$) design. Again, the data provide directional support and partial statistical support for both hypotheses, and the more general claim that first-round investments are sensitive to group size. ANOVA reveals a main effect in k/N ($F(1,176)=3.73$, $p=.055$), which is consistent with H_1. There is also a marginally significant interaction effect ($F(1,176)=2.72$, $p=.10$), and no main effect in N at $p<.1$. Both results lend support to H_{2A}(b) and H_{2B}(b). However, none of the pairwise comparisons testing simple effects in N at either $k/N=2/5$ and $k/N=4/5$ yield significant results at $p<.1$. Overall, we thus conclude that there is directional

[6] Incidentally, since H_1 is at least supported at $N=5$, we can also reject the hypothesis that subjects invest the same amount in the first round regardless of k and N.

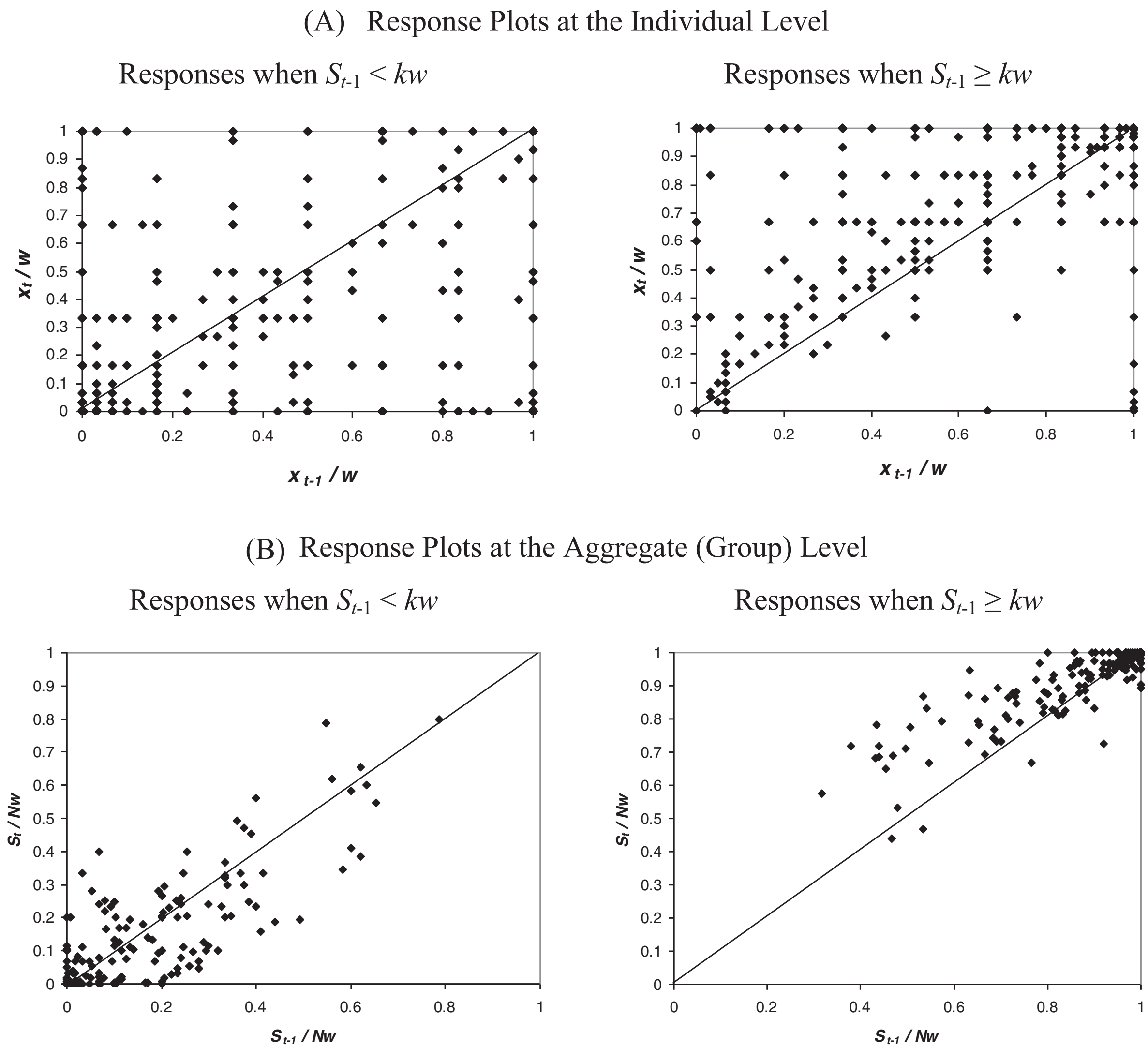


Fig. 3. (A) Response plots at the individual level and (B) response plots at the aggregate (group) level.

support for all the statements in H_{2A} and H_{2B}, while ANOVA provides partial statistical support for them mainly through the marginally significant interaction effects.

4.3.2. Dynamics

Fig. 3A and B is one-lag response plots at individual and aggregate levels, respectively. Consistent with hypotheses H_3 and H_4, when $S_{t-1} \geq kw$, players seem to be optimistic and increase their investments in round t (observe that most points in the right-hand figures are above the diagonal). When $S_{t-1} < kw$, players believe that they face a trend towards the no-investment equilibrium and trade off between investing less to cut their losses (observe that most points in Fig. 3B are below the diagonal) and investing some amount to "cue" others to "resist" the trend. Hence, responses fluctuate more when the trend is decreasing, a fact supported also by Fig. 1A and B.

To test hypotheses H_3 and H_4 more rigorously, we classify and count responses according to the hypothesis premises and predictions, producing Table 2 as a result. Chi-square tests support all the statements in both hypotheses at the $p < .05$ level and in fact at the $p < .0001$ level in all but one case. The higher p value in the test of statement (b) of H_4 corresponds to the fact that a lot of the points in the $S_{t-1} < kw$ panel of Fig. 3A are below the diagonal, indicating that subjects have a tendency to "resist" convergence towards the inefficient no-investment equilibrium. In fact, even the $\chi^2(1)$ for testing H_4(a) is smaller than that for testing H_3(a), which also suggests a tendency to "resist" making the inefficient equilibrium investment of zero.

Next, we check whether H_5 is supported. We first code each group in each condition such that all groups that converge towards the no-investment equilibrium are coded as type 0, while all groups that converge towards the full-investment equilibrium are coded as type 1. Then, for each group, we define a dummy variable, d, which is equal to 1 if $S_{t-1} \geq kw$ and

Table 2
Individual response counts.

		Condition in round $t-1$ (related hypothesis)			
		$S_{t-1} \geq kw, x_{it-1}=1$ (H$_3$(a))	$S_{t-1} \geq kw, x_{it-1}<1$ (H$_3$(b))	$S_{t-1} < kw, x_{it-1}>0$ (H$_4$(b))	$S_{t-1} < kw, x_{it-1}=0$ (H$_4$(a))
Response counts	$x_{it} < x_{it-1}$	56	37	220	NA
	$x_{it} = x_{it-1}$	2351	68	95	1214
	$x_{it} > x_{it-1}$	NA	368	79	72
Chi-square test for related hypothesis	$\chi^2(1)=2{,}188$, $p<.0001$	$\chi^2(1)=146.2$, $p<.0001$	$\chi^2(1)=5.371$, $p \approx .02$	$\chi^2(1)=1{,}014$, $p<.001$	

Note: For the appropriate chi-square test for H$_3$(b), we collapse the counts of $x_{it} < x_{it-1}$ and $x_{it} = x_{it-1}$; for the appropriate chi-square test for H$_4$(b), we collapse the counts of $x_{it} > x_{it-1}$ and $x_{it} = x_{it-1}$.

0 otherwise. A chi-square test of the distribution of type and d among groups shows that if $S_{t-1} \geq kw$ predicts the type of convergence ($\chi^2(1)=19.85$, $p<.0001$); in fact, only three out of the 30 experimental groups do not satisfy the predicted relation. Hence H$_5$ is supported.

Hypothesis H$_6$ receives directional support by the percentage of convergence to the full-investment equilibrium in the relevant conditions, as shown in Table 1. Chi-square test yields significant effects for comparison between (5,2) and (5,4) ($\chi^2(1)=8.57$, $p=.0034$), as well as for comparison over (10,2), (10,4), and (10,8) ($\chi^2(2)=18.00$, $p=.0001$); both provide additional statistical support for H$_6$. Hypothesis H$_7$ is supported by comparing the percentages of convergence to full-investment equilibrium in (5,4) and (10,4), for which chi-square test yields significant effect ($\chi^2(1)=8.57$, $p=.0034$). We conclude that the convergence hypotheses are directionally and statistically supported by our data.

5. Conclusions and future research

In this paper, we investigate investment decisions and the associated coordination problems in a market with network externalities. For this purpose, we adopt an experimental approach with behavioral hypotheses and modeling. Specifically, we examine how critical mass (which arises naturally in the presence of network externalities) and group size influence investment decisions. To single out the effects of these two factors, we have kept out other complicating factors, so that there is only one type of investment and agents are homogeneous in our experimental setting.

We first establish a multi-person game with desirable properties for our objectives. The design of our game resembles the provision point public goods game but there are also substantive differences – such as free riding being impossible in our case – that cater to features of network externalities. We propose a number of behavioral hypotheses regarding first-round investments, dynamics, and equilibrium convergence, for our experiment to test. We then present experimental results showing that the Pareto optimal equilibrium is sometimes, but not always, attained. This observation is common with previous studies on coordination game.

Our hypotheses on first-round investments and equilibrium convergence are supported by the data. They reveal a subtlety about strategic uncertainty that has not been picked up by previous studies on coordination problems. What previous literature (e.g. Croson and Marks, 2000) suggests by analogy, and what we have observed, is that first-round investments and the likelihood of convergence towards full-investment equilibrium are non-increasing in critical mass holding group size constant. Previous literature (e.g. Van Huyck et al., 2007) also suggests a weakening of coordination to reach the Pareto optimal equilibrium as group size increases, but our findings present a more complicated picture. While an increase in group size (controlling for either the critical mass or the minimum average investment needed to the reach the critical mass, as stated in our hypotheses) is expected to lead to an increase in strategic uncertainty, if investments are inclined towards the no-investment equilibrium when group size is small, they may actually increase as it becomes bigger. This is because, in this case, when subjects want to reduce opportunity cost in a more uncertain environment, in which they are less sure that others would *not* try to reach the Pareto optimal equilibrium, they tend to invest more when group size increases. This mechanism is indeed evident in our data both in the short run (the first-round investments) as well as the long run (equilibrium convergence).

In accordance with experimental results on order statistic games such as VHBB, convergence to the efficient equilibrium is sensitive to what happens in the first round; in our case, it is clearly predicted by whether or not the total investment reaches a critical mass in the first round. Thereafter, S_t, the total investment in a round, becomes a "market reputation" signal with positive feedback characteristics as a side effect of the reinforcement learning process that we posit. As a result, S_1 becomes important in determining the direction of convergence. This finding leads us to suggest a managerial insight regarding the initial market share of a newly launched product or newly introduced technology with network externalities: if such a product/technology is to succeed in capturing the market, the network size must be "jump-started" past a critical tipping point at which payoff for joining the network is greater than the cost of joining (in time, effort, and resources). Our results also offer the insight that, even if total investment fails to reach critical mass, if at least a portion of the investors realize that full investment is Pareto optimal, then convergence towards the no-investment equilibrium will not be smooth but will meet with noisy "resistance". This is reflected in Fig. 1A and B, the response plots Fig. 3A and B, as well as the smaller $\chi^2(1)$ statistics in the right columns of Table 2 relative to those in the left columns.

As an experimental setup, the game we have used can be modified and generalized in several directions. For example, competition between products with externalities can be investigated by allowing subjects to invest in more than one product, so that product m has critical mass $k_m w$ and offers network externalities gain $x_{i,m}\sum_{j\neq i}x_{j,m}/k_m w$ to subject i with investment $x_{i,m}$ on m. Heterogeneity can also be introduced. Each player, i, may have her own endowment, w_i, and parameter, k_i, that can be different from other players' endowments and parameters, thus allowing for heterogeneity in critical mass. It may also happen that the "intrinsic" utility the player has from investing in the product is β_i times the network externalities she gains from other players' investments, so that her payoff is:

$$\pi_i = w_i - x_i + \frac{x_i\left(\beta_i x_i + \sum_{j\neq i}x_j\right)}{k_i w_i} \quad \text{with} \quad \forall i \quad w_i \geq x_i \geq 0.$$

Note that $\beta_i = 1$ for all i in the present setup; the economic model put forward in Section A.1 in Appendix provides an example for when this might be a reasonable assumption. Another direction for extension is that players are given a one-off endowment at the beginning of round 1 that is not replenished thereafter, and any investment in round t becomes a sunk investment that persists in the calculation of the payoff in every round after t. This brings the experiment nearer to the durables market, while the present setup can be interpreted as approximating the market of a rental or subscription product with network externalities characteristics, such as a new telecommunications service (see also Section A.1 in Appendix A) or new software products (e.g. multiplayer online games) operating on a per period subscription model.

Acknowledgements

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Appendix A. Supplementary materials

Supplementary materials associated with this article can be found, in the online version, at doi:10.1016/j.jebo.2010.08.017.

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