Invariance failure under subgame perfectness in sequential bargaining

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Abstract

A basic property of any normative theory of decision making – individual or group – is its invariance under the theory's own equivalence specification. Growing evidence from experimental studies in several areas of game playing indicates that the game-theoretic notion of strategic equivalence is systematically violated in the behavioral arena. The present study expands the design of previous studies of bilateral bargaining by including a third party and a new trading rule – modifications which induce behavioral patterns that reject equivalence under subgame perfection. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

Suppose I believe that expected value is a reliable guide for predicting your behavior under risk and I present you with a choice between receiving 1001 dollars for sure or playing a binary lottery consisting of winning $2002 + \epsilon$ with probability of one-half and gaining nothing with probability one half (ϵ is small). You pass this opportunity, even though the expected value of the lottery exceeds 1001. How come? Under expected value hypothesis, given a decision tree, substitutions of lotteries for others with same expected value are permissible. Thus, a substitution of 1001 for the left-hand term in the sum of the two lotteries

$$
\begin{array}{ccccc}
0 & & 2002 & & 0 & & \epsilon \\
{}_{.50}\searrow & \nearrow_{.50} & & + & {}_{.50}\searrow & \nearrow_{.50}
\end{array}
$$

shows that had you really been an expected value maximizer you would not have passed the extra opportunity. This is a failure of invariance, a failure with respect to expected value theory as a descriptor of your choice under risk. Invariance originated in 19th century mathematics and applied by Felix Klein to classify the hierarchy of classical geometries as the study of invariants of the classical groups. An invariant for one geometry, such as length in the Euclidean geometry, may fail under groups more general than the Euclidean group of transformations, for example the similarity group. The essential point is that invariants are relative to a particular theory. Thus, the invariance failure alluded in the title is relative to game theory, and the specific common knowledge information enveloping the experimental procedure employed here.

Of course, we all know that the type of *gednake* experiment described above gave birth to expected utility formulation of risk preferences, and subsequently, following the experimental refutation of expected utility theory, to the development of nonlinear expected utility theories and related descriptive theories (e.g., Prospect Theory of Tversky & Kahneman, 1992). The significance of the invariance failure relative to expected utility of choice (say, by the stable evidence of the preference reversal phenomenon Slovic & Lichtenstein, 1983) is that this theory is grounded on axioms which are the hallmark of rational choice.

Similarly in bilateral bargaining, recent research has shown that fixed sums appearing as outside options in Rubinstein's two-person sequential

bargaining paradigm are important factors in contributing toward subjects' rational behavior (e.g., Binmore, Morgan, Shaked & Sutton, 1991). But if bargainers in the extensive form game play subgame perfect strategies, for the game trees we shall explore below, standard game theory would suggest that equivalent bargaining options are substitutable. Results show that this is not the case. Rather, we provide experimental evidence in support of a hypothesis that bilateral bargaining in the classical Rubinstein's paradigm is fraught with risk, and that subgame perfectness is much less than a perfect guide in predicting bargainers' behavior.

Although the setup we shall explore – the comparison of three-person to two-person *sequential* bargaining – is similar to comparisons of three-(and more)-person to two-person *ultimatum* 'bargaining' (Roth, Prasnikar, Okuno-Fujiwara & Zamir, 1991; Güth, Marchand & Rulliere, 1997; Güth & van Damme, 1998), the motivations underlying these two endeavors are fundamentally different. Whereas in the ultimatum field the issue is the enhancement of greedy action on the part of the person who in theory has all the power but refrains from using it (due to the high risk of loss), the same theory of rational bargaining as it is applied to the framework of Rubinstein bargaining appears to be cognitively opaque to human players. This paper shows that where players have no clue to guide them in rational action they can be easily swayed to act irrationally, and still, perhaps, believe in their virtue.

Of course, we have by now plenty of experimental evidence from the finitely iterated Prisoner's Dilemma game, the Centipede game (e.g., McKelvey & Palfrey, 1992), and the two-person sequential bargaining game (e.g., Ochs & Roth, 1989) that violates subgame perfectness. Whereas this body of experimental work suggests that subjects do not use the backward induction process underlying the derivation of the subgame perfect solutions to these games, particularly when the subjects are inexperienced or the number of moves in the extensive form game is relatively large, our procedure of testing the implications of subgame perfection – which invokes the idea of equivalence between classes of games – is entirely different.

Rubinstein bargaining framework, with and without complete information, seems to us a more congenial abstraction of bargaining scenarios than those experimented within recent years, where an artificial stopping rule is clearly known to the subjects, and failures of subjects to follow rational strategies in those cases may have only limited bearing on real bargaining. The simple but confusing notion of backward induction due to the finite stopping rule does not appear as prominently in the games we experiment

with. As a result it is not at all clear what characteristics of playing finite games have to bear on playing Rubinstein bargaining games. Moreover, recent theories of learning (Roth & Erev, 1995) and equity (Bolton & Ockenfels, in press) have very little to say on the Rubinstein scenario, as they were attempts to deal with aberrations of rationality in an essentially finite game playing.

2. The paradigm

A *standard* discount-rate bargaining game consists of two players who alternate offers in order to reach an agreement as to the division of a given sum of money (normalized to unity) that they jointly possess. The game terminates at a discrete time t when, and only when, an offer is accepted. It is assumed that the utility of a part x of the sum is represented by $x\delta^t$, where δ is a personal discount rate between zero and one. This is essentially the game proposed by Rubinstein (1982) and studied experimentally, among others, by Weg, Rapoport and Felsenthan (1990).

The *ultimatum restriction* of a standard discount-rate bargaining game differs from it only in the rule governing the generation of offers. Whereas in a standard game the two players alternate in making offers, the ultimatum restriction game allows only one player the right to generate offers and the other player the right to either accept or reject such offers.

We now devise an alternative to each of these two bargaining games. First, a *standard three-person extension* consists of *three* players who need to share a joint sum. Player 1 starts bargaining with one of the other two players, say player 2, about sharing the sum between them (players 1 and 2). They follow the same rules as in the standard two-person game except that player 1 has the option, called an outside option, to refuse an offer and in the following bargaining period to start, if he so desires, the same type of bargaining with player 3 as was originally started with player 2. Thus, when it is time for player 1 to make an offer, she is permitted to make an offer to any of the other two players, but the latter can only accept, reject, or counter-propose. [1] For the purpose we have in mind, we demand that the discount rates of players 2 and 3 are equal.

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[1] We could have chosen to allow players 2 or 3 to opt out of the bargaining with zero payoffs, with no change in the theory. However, this seems less natural.
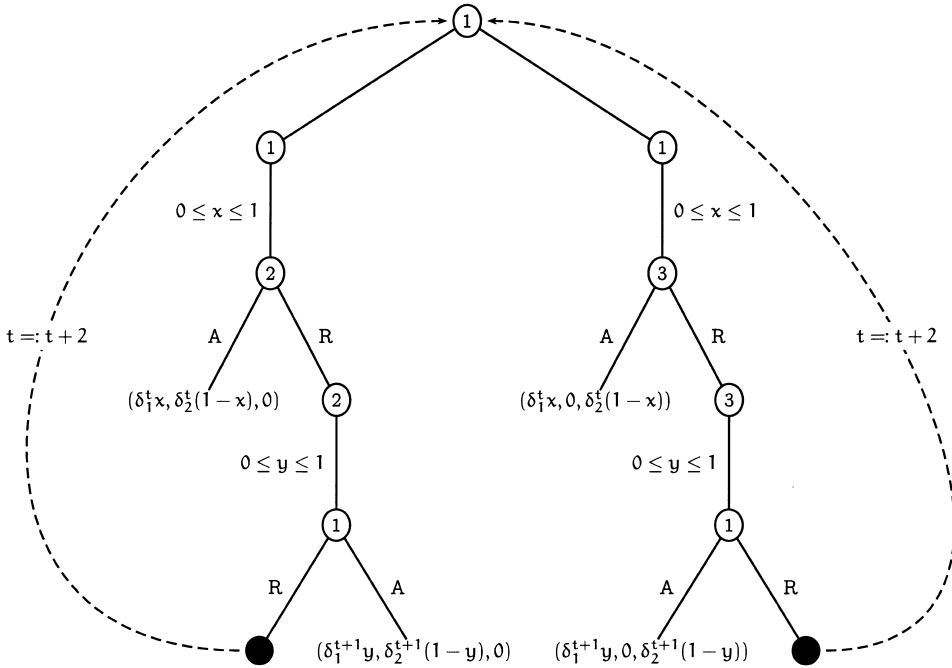
Fig. 1. Standard three-person extension game. The 2-person standard game is obtained by deleting the root and left (or right) main branch. The ultimative forms are obtained from their respective standards by clipping all branches starting from offer nodes belonging to players other than 1 and looping back to the root from R-edges thereby making $t = t + 1$.

The second game alternative, the *ultimatum three-person extension*, is the same as the ultimatum restriction of the standard two-person game but with the player switching mechanism provided by the standard three-person extension. Fig. 1 depicts the games.

2.1. Solutions

How *should* bargainers play these games? For them to consider any Nash equilibrium is tantamount to allow for noncredible commitments, which are the part of the Nash equilibria that subgame perfectness was designed to eliminate. For it is clear that any division of the sum in *any* of the games is supported by Nash strategies. The fact that noncredible commitments are not possible allows for a refinement, namely, for a selection of some special Nash equilibrium. Being aware correctly, if not precisely, of such effects is the hallmark of rational behavior in these cases.

To construct the subgame perfect Nash equilibrium (a Nash refinement), we consider the games in the order we have described them. In the two-person standard discount bargaining, game player 1 thinks she should be given a proportion x. To be consistent, player 2 then obtains $1 - x$. Now x is subject to the restriction that after two periods of bargaining without an agreement the result of bargaining during the induced subgame should be the same as supported by the original game. Hence, x obeys these equations:

$$x = 1 - \delta_2 y,$$

$$y = 1 - \delta_1 x,$$

where y is the payoff to player 2 had she or he been given the opportunity to start the bargaining, and the δ's are the individual discount rates. Solving these two equations shows that

$$x = \frac{1 - \delta_2}{1 - \delta_1 \delta_2}. \tag{1}$$

Note that what is obtained is nothing but the subgame perfect equilibrium allocation. This is implicit in the argument regarding the outcomes in sub-games. A full justification for this simple-minded accounting is given by Rubinstein (1982) and Sutton (1986).

Consider now the ultimatum restriction of the standard discount rate game. Since player 2 is precluded from making offers, player 1's take, x, should satisfy
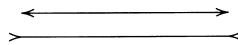
$$x = 1 - \delta_2(1 - x).$$

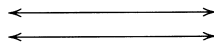When player 2 is even slightly impatient $(\delta_2 < 1)$, x must be 1.

We next turn to the standard three-person extension. Once player 1 chooses a co-bargainer, the resulting subgame is a game with a voluntary outside option (Binmore, Shaked & Sutton, 1989; Weg, Zwick & Rapoport, 1996). Since it is assumed that the discount rates of players 2 and 3 are the same, the rational choice to whom to direct an offer results in equivalent subgames. But taking the outside option cannot bring a better outcome to player 1 than continuing the bargaining with the same player. It follows then that player 1's payoff in the standard extension is the same as player 1's payoff in the two-person standard game. A similar conclusion is obtained regarding the ultimatum three-person extension games. Underly-

ing this argument is the assumption that both the rules of the game and the individual payoffs are common knowledge, and that the buyers' tolerance to aggressive offers by the seller are the same. This assumption is commonly made by game theory that, in our case, characterizes the players by their payoffs and discount factors. If this assumption is violated (e.g., by the seller suspecting that different buyers have different tolerances to her aggressive offers), then invariance may no longer hold, and the seller may incline to switch between buyers. But how could this assumption be violated? Unless player 1 is clairvoyant, she has no rational basis to make a distinction between her opponents and thus playing against two nominally identical players is the same as playing against one representative, at least regarding the first offer!

Note that the equivalence between games and their extended (three-person) versions is independent of the nature of the solution in the corresponding (two-person) standard/ultimatum ones. This follows from the argument where the precise payoff in the latter games does not appear at all. In particular, regardless of the rational sharing principles bargainers follow while playing standard/ultimatum games, if they follow minimal rationality as we have expounded, they must agree on similar outcomes for their extended versions. Otherwise, they follow principles to which game-theoretic considerations are blind. In analogy, most people judge the straight lines in B to be of equal length, but judge the lower line in A as longer than the top line in A.

A

B

However, rational theory of measurement would not support it.

This claim of solution independence is not essential to the primary goal of this paper: that subgame perfect Nash equilibrium does not play a significant role in discount bilateral bargaining. One needs only to submit to the assertion that if players adopt subgame perfect Nash equilibrium, their

strategies in the three-person should correspond to their two-person counterparts. [2]

Another point of view, to be exploited later, is the idea of bargaining between *classes*, which takes the role of bargaining between the players. This is so, for the players are characterized by their impatience, and individual players have no means of differentiating between representatives of a given class. The two players 2 and 3 are representatives of the same class, and player 1 makes up her own class. It is now clear that the three-person games, seen as games between equivalent classes, are in reality isomorphic to the two-person games, standard and ultimatum types, respectively.

The choice of the three-person bargaining under the particular rules we have adopted is guided by the special requirement to provide a natural outside option to at least one of the players. The usual method of providing outside options is to allow a player to resign the bargaining and to consume a given nominal sum. Thus, in our case, player 1 will be provided with an outside option of $\delta/1 + \delta$ one period later. It was shown by Binmore et al. (1989), and to a lesser degree by Weg et al. (1996), that subjects attend positively to the outside options.

We conclude that regardless of the solution adopted by the players – subgame perfect or not – the weak assumption of invariance implies that payoffs in the two-person games should be the same as those found in their three-person equivalents.

3. Method

Two hundred and forty male and female undergraduate students participated in the experiment, each in a single session that lasted approximately an hour and a half. Subjects were recruited through classified advertisements placed in the campus newspaper, as well as through class announcements and billboard advertising, promising monetary reward contingent on performance in a bargaining study.

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[2] If, however, they choose the more general Nash equilibrium, there arises the problem of co-ordination and selection from a myriad of possibilities and playing a game might last longer. But regardless, in this case it means that subjects choose to ignore strategic advantage, either out of ignorance or out of choice.

3.1. Design

The bargaining scene was couched in economic language to elicit better realism. In all games there was one seller and either one or two buyers whose goal was to divide a surplus of $10 under a *common* discount rate with the seller opening the bargaining.

The experiment used a two (one vs. two buyers) by two (standard vs. ultimatum game) by two (1/6 vs. 2/3 discount rates) by nine (iterations) factorial design (see Fig. 2). The first three factors were between subjects, and the last factor within subjects. The first factor, namely, the number of buyers, was equal to either one (two-person game) or two (three-person game). The second factor was the (trading) rule governing the generation of offers. The third factor was the common discount rate, whose value was commonly known. The fourth and last factor of the design was iteration of the game: each subject played a total of nine rounds – the first six for practice and the last three for actual pay.

During the practice rounds, each subject played each role the same number of times. In contrast, subjects were randomly assigned fixed roles during the cash games, subject to the requirement that they had never faced their partners during the practice games. The purpose of these practice rounds was to acquaint the participants with the game by experiencing the roles of both buyer and seller.

3.2. Procedure

Upon arrival at the laboratory, subjects were handed written instructions which contained the description of that session's bargaining rules (see Appendix A).

The instructions presented a scenario in which a student seller could sell a used textbook to a single (or one of two) student buyer(s). The sellers' only profit was the negotiated selling price of the book. A buyer was described as given $10 to spend for the book. The book might be bought for more than $10 from the bookstore. The negotiation rules and the choice of a common discount rate were also described. The particular set of instructions given was determined by the session, classified according to the first three factors of the experiment (Fig. 2). It was assumed that by publicly giving the same instructions to every subject all details become common knowledge.

Subjects interacted via computers. It was emphasized that they would not know their co-bargainers' identity, nor would their co-bargainers know

Experiment

Fig. 2. The experimental design.

theirs, and that these identities would not be revealed after the session was completed. Proposals, acceptances, and rejections were transmitted through computer terminals. A price proposal was given meaning in terms of net profits, and proposers could verify their proposals before submission. All information exchange among the players in the same play was common knowledge. No other communication was allowed.

It was emphasized that at the end of the session the subjects would be paid discreetly. In practice, negotiations broke off if they entered the 14th period. This occurred only three times and then only during the practice plays.

4. Results

We begin by presenting a summary of the cash play results (from three trials) supporting the main conclusions of the experiment. These are followed by tables presenting the main statistics and statistical justifications of our conclusions:

- The first period demands of player 1 (Seller) in the extension games are significantly higher than those obtained in their corresponding two-person games. The mean first period demand is 6.84 (0.884) for three-person games compared to 6.20 (0.698) for two-person games. [3]
- However, this effect is not uniform. It depends on whether the bargaining is standard, where the effect is very clear, or it is restricted to ultimatum bargaining, where the effect is mixed.
- Although ultimatum games ascribe higher power to the seller she does not use it: the mean first period price is 6.55 (0.955) for standard games vs. 6.49 (0.754) for the ultimatum games. Note that these games command prices far lower than game theory [4] would lead us to expect.
- A smaller discount rate does not necessarily result in higher prices, which is theoretically expected. The mean first period price is 6.67 (0.833) for the 1/6 discount rate and 6.37 (0.863) for the 2/3 rate. This is only half the difference noted in the previous comparison. A closer look reveals that this mild difference is a result of different behavior in ultimatum and standard bargaining, the former masking the latter's distinct differences between offers due to discount rates. Nonetheless, we note the overall extreme conservatism regardless of the type of bargaining game.
- Playing for cash results in very little alternation of bargaining partners.
- We remark that, in general, the practice games were more volatile. This is shown by the length of bargaining needed to reach agreement. Fig. 3 shows a weak but distinct monotonicity in the duration of bargaining

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[3] Standard deviations appear parenthetically, all based on 36 independent observations.

[4] This is invariably game theory which does not ignore relevant information in formulating its recommended behavior. It is also game theory which is in no way influenced by irrelevant information.

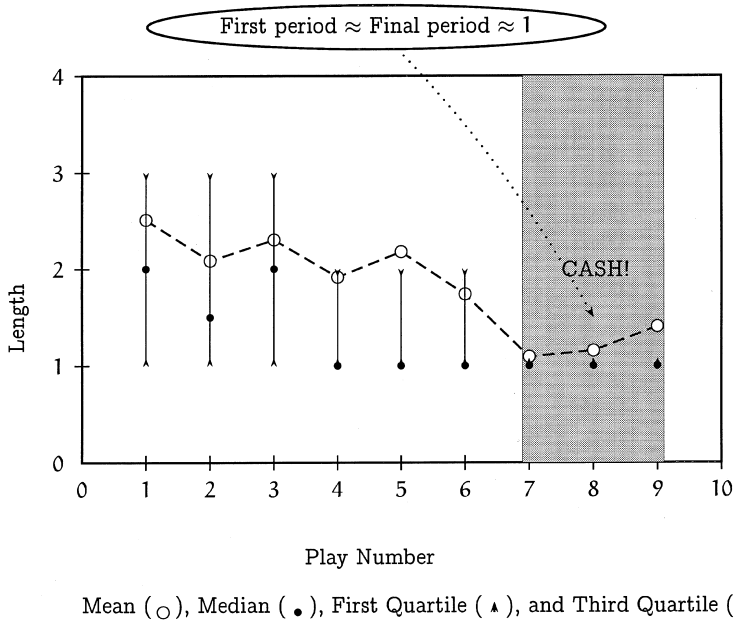Mean (○), Median (•), First Quartile (▲), and Third Quartile (▼)

Fig. 3. Four point summaries for period-at-termination by play number.

with serial position of a play. On the average, cash plays took half the time to complete. In addition, 87% of all cash plays ended with the first offer accepted, 96% ended within the first two periods, and 98% within the first three periods. As a result, one can see in Fig. 3 that the first and third quartiles coincide for later plays. We conclude that the final offer is essentially the same as the first offer, and deserves no further discussion. [5]

We next present statistical justifications for our main conclusions. To investigate whether behavior changes during the three cash playing games, we examined each seller's changes in the first period price offers. There are two (algebraically) independent changes. [6] This set of pairs is classified according to the main classification scheme of the experiment (see first three levels in

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[5] Here we make a judgment by rejecting the notion that 13% of rejected first offers should color our general conclusions. In any case, the reader may bear in mind that perhaps a small proportion of subjects are not duly represented in our discussion.

[6] To be concrete, we take the difference between the first play and the second and between the second and the third.

Fig. 2). The distribution of pairs appears to be independent of any of the classification (no significant differences in multivariate analysis of variance, see Table 1). On the other hand, the means of these pairs, $(-0.091, -0.108)$ indicate that the distribution from which they are sampled has non-zero mean ($F = 6.437$, $d.f. = 2, 188$, $P < 0.01$). We conclude that, on the average, sellers increase their first period price demands across iterations, though not in a meaningful way.

Next, we investigated the manner in which the mean of the first period price demand by the seller (Table 2) is affected by the main experimental conditions.

Here, we find through analysis of variance (Table 3) that except for a difference in means due to two-person vs. three-person games (main factor C) and its interaction with standard vs. ultimatum game type (interaction effect AC), no other effects are significant at the 0.01 level. The AC interaction

Table 1
Multivariate analysis of variance of vector of differences in first period demand[a]

Factor	F-stat	$d.f.$	Sig.
Game type (A)	0.837423	2, 174	ns
Discount (B)	0.588728	2, 174	ns
Game size (C)	1.75771	2, 174	> 0.05
AB	2.17562	2, 174	> 0.05
AC	2.15017	2, 174	> 0.05
BC	0.774659	2, 174	ns
ABC	1.66901	2, 174	> 0.05

[a] Derived F-statistics are precise.

Table 2
Mean first period cash price demands by the main classification variables ($N = 12$ sellers per cell)

	Discount	Game size		Overall mean
		2	3	
Standard	1/6	6.315	7.391	6.853
	2/3	5.587	6.904	6.245
Mean		5.951	7.148	6.549
Ultimatum	1/6	6.633	6.333	6.483
	2/3	6.246	6.748	6.497
Mean		6.439	6.541	6.490
Overall mean		6.195	6.844	6.520

Table 3
Analysis of variance for first period mean price

Factor	F	d.f.	Sig.
Game type (A)	0.162	1, 88	ns
Discount (B)	4.074	1, 88	$0.01 < P < 0.05$
Game size (C)	19.438	1, 88	$P < 0.01$
AB	4.468	1, 88	$0.01 < P < 0.05$
AC	13.845	1, 88	$P < 0.01$
BC	3.134	1, 88	$P > 0.05$
ABC	0.907	1, 88	ns

effect is not of the strong type (i.e., crossing graphs) as we can conclude from an inspection of Fig. 4, which displays the empirical distributions of the mean first offer each seller gave (over the three games played). One can see very clearly that the upper two subfigures as well as the lower right show distinct uniform gap between the distribution of mean first price offers when faced with one buyer as opposed to two buyers. The two-buyers graph is shifted to the right indicating higher price demands. Although the mean offer is higher for the single buyer in the left lower subfigure, the difference is of no consequence. It stems from few demands around the equal split. By the 60% split the two distributions are the same. As another reassurance, we have tested the difference between the distributions for each subfigure using a very powerful non-parametric test suggested by Epps and Singleton (1986). [7] The only non-significance effect is exactly where the visual inspection suggests it is.

At a less stringent level of significance (5%), we find that the discount rate (main factor B) is an effective determinant of the first period price offer. This is even clearer upon noting that this effect is dependent on the type of bargaining (standard vs. ultimatum) evidenced in the AB interaction effect (also noted in Table 2). This corroborates the first four statements made above.

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[7] This test compares characteristic functions (CF) of two distributions. The values of the CF are needed only at two points, $t = 0.4$ and $t = 0.8$, provided the random variables are normalized by a scale parameter, similar to the semi-interquartile range, estimated from the combined sample. The original sample provides a sample from the random variable $\exp(iXt)$ whose expectation is the CF. Therefore, the mean of the sample is an estimate for the CF. The statistic actually used is very similar to T^2 of Hotelling, but it relies on large numbers and therefore on the central limit theorem to obtain a statistic distributed approximately as $\chi^2(4)$. For small samples a correction based on a simulation study is provided. For a precise description see the source.
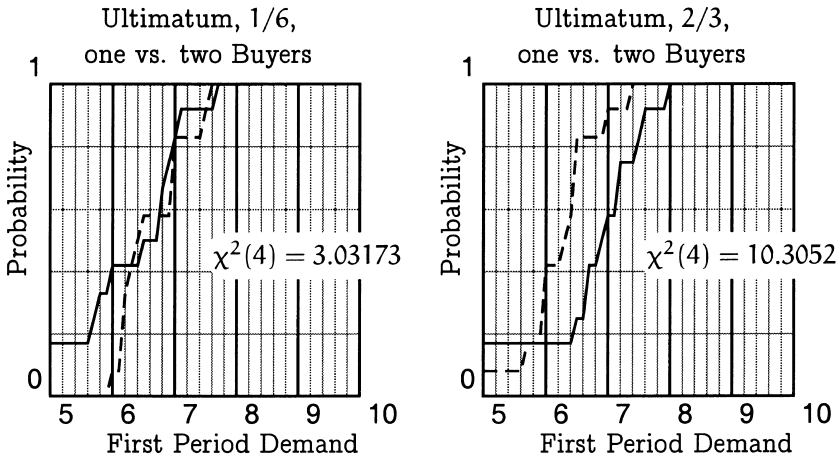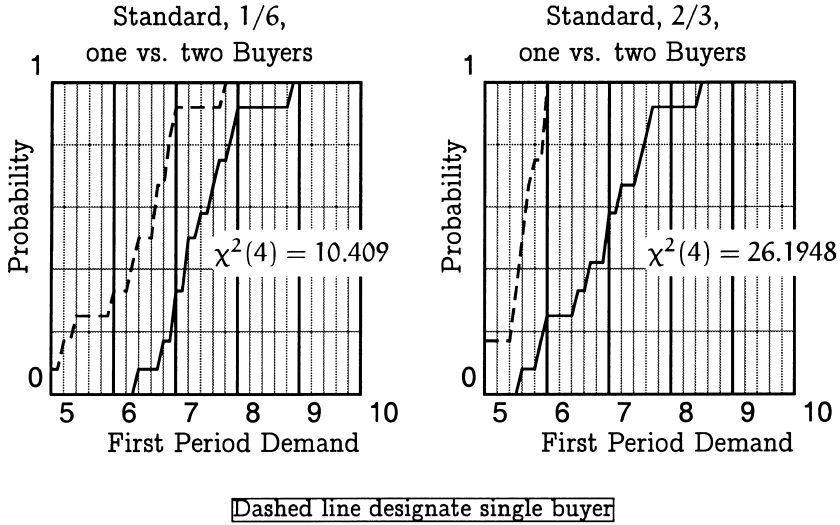
Fig. 4. Cumulative distributions of first period demands.

Table 4 presents the frequency distributions of buyer switch patterns for both practice and cash trials. It is clear that the practice games present an opportunity to explore the bargaining space, and that this activity is much more restrained during cash games. This can be seen by comparing the number of switches per play during the two phases of the experiment.

Table 4
Frequency of buyer switch patterns[a]

Condition	Ratio	Switch patterns (frequency)	switch/play
Ultimatum — Practice	1/6	+ (42), +o (12), ++ (3), +oo (2), +o+ (7), ++o (1), ++o+ (1), +oo++ (1), ++++ (1), +oo++o (1), +++ooo (1), +oo+oo (1)	40/72
	2/3	+ (41), +o (14), +oo (5), +o+ (4), +++ (1), +o+o (2), +++o (2), +o++o (1), +o+oo (1), +o++o (1), +o++o+ (1)	45/72
Ultimatum — Cash	1/6	+ (34), +o (1), +o++++ (1)	3/36
	2/3	+ (31), +o (2), ++ (3)	2/36
Standard — Practice	1/6	+ (55), +o (9), +oo (3), +o+ (1), ++o (1), +oo+ (1), +o+o (1), ++++ (1)	20/72
	2/3	+ (45), +o (17), ++ (3), +oo (3), +o+ (2), +ooo+ (1), +o+o+o (1)	31/72
Standard — Cash	1/6	+ (32), +o (3), +oo+o (1)	6/36
	2/3	+ (34), +o (1), ++ (1)	1/36

[a] The +'s are for one buyer and o's for the other. Such labels are counted when seller is in a position to offer. Thus, for *Standard* games it is every other period and for *Ultimatum* games, every period. Example: the pattern +o+ (in ultimatum, 1/6, practice plays) indicates that in 7 plays (which lasted 3 periods) sellers switched to an alternative buyer immediately after their rejected offers.

5. Reconciliation with previous studies

The results of the present experiment are best interpreted in light of previous research. There are several published experiments which might give the impression that subjects consider recursively (albeit imperfectly) possible offers and counter offers in unlimited time discounted bargaining. An attractive interpretation is that subjects strive for rational behavior in the manner stipulated by subgame perfectness (Selten, 1975; Rubinstein, 1982). We describe in detail one experiment by Binmore et al. (1989), which supports subgame perfect playing, and another by Weg et al. (1990), which does not.

Binmore et al. (1989) set out to show that the conventional wisdom of evenly sharing the leftover after accounting for outside options (obtained if bargaining fails to reach an agreement) is not always a reliable predictor of behavior. For this, they exploit the voluntary exit bargaining procedure as follows. In addition to the alternating pattern of offers and counter-offers as in the standard game, a player (1, or 2) may access a given sum s_i (an outside option) when and only when he rejects an offer. In this way, the bargaining terminates with these set of values. Of course, the size of the s_i is such as to make the bargaining relevant. One can show that the unique [8] subgame perfect partition equilibrium for this game is given by the solution of the equation:

$$\min(\max(1 - \delta_2 + \delta_2\delta_1 x, 1 - \delta_2(1 - s_1)), 1 - s_2) = x$$

(which, because the left side is continuous, is easily seen to exist) as the payoff to player 1. It follows that if

$$\frac{1 - \delta_2}{1 - \delta_1\delta_2} > 1 - s_2, \tag{2}$$

then we can interpret this case as if bargaining stops with players taking their side option, and if

$$1 - \delta_2(1 - s_1) < \frac{1 - \delta_2}{1 - \delta_1\delta_2} \leqslant 1 - s_2, \tag{3}$$

then normal bargaining takes place as if outside options are irrelevant. If we assume that $\delta_1 = \delta_2 \approx 1$, then time does not exert too much pressure and the

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[8] We assume that at least one discount rate is less than 1.

bargaining outcome is close to fifty–fifty. This is easily predictable from the almost symmetric environment as well as from the formula. But if Eq. (2) obtains, player 2 can force his outside option. And this, in fact, is what Binmore et al. (1989) show. [9] They take a case where player 1's outside option is 0 and player 2's can be either more than half or less than half the pie. The discount rates are high. Bargainers behave exactly as predicted by subgame perfectness and not by some arbitrary Nash strategic equilibrium or by Nash (1950) bargaining solution.

On the other hand, Weg et al. (1990) set out to observe behavior in the classical Rubinstein case. They experimented with equal as well as unequal discount rates. The general conclusion reached is that for equal discount rates, subjects split fifty–fifty with an advantage to player 1, *regardless* of how impatient they are. This is obviously not what one would expect from recursive reasoning. The present experiment aims to substitute into the design of Binmore et al. (1989) risky outside options in the sense that players need to assess their value on their own. Normally, outside options given as fixed sums are thought of as present value resulting from a given move. However, this may not be the way subjects perceive them.

Following these two studies, Weg et al. (1996) conjectured that factors not inherent to the essence of bargaining may, under favorable conditions, lead one to the conclusion that subjects are monotonically subgame perfect. Given the heuristic interpretation of outside options in this bargaining paradigm, this conclusion might be considered impressive. But in natural settings it is often the case that opting out may lead to some uncertain game-oriented (as opposed to probabilistic) consequences.

How do we, then, reconcile the performance of the subjects in the current experiment? Recall that ultimatum games prescribe no profits for buyers: the seller demands the maximal price. Also, for standard games, Eq. (1) reduces to $1/(1 + \delta)$, hence price is monotonically decreasing with discount rate. For $\delta = 2/3$ the normalized price is 0.60. Hence, inspecting Table 2, one might conclude that contrary to our statements, subjects are doing fairly well. But for a discount of 1/6 the expected sale price is $6/7(\sim 86\%)$, which is far higher than the one seen in the table. One might try to explain the conservative price offers in playing ultimatum and high discount standard games by an appeal to justice considerations. Had this been the whole story one should

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[9] We do not take the trouble to sort all the cases, except to convince the reader that as a test of rational subgame perfect behavior it is very weak.

not find (but we do!) higher prices for three-person than for two-person games. These prices are also higher than those obtained in playing ultimatum two-person games. [10]

Choosing an action relies on its anticipated consequences, and the prediction of behavior might benefit from a clear distinction between the *formal* and the *mental* representations of a situation. As applied to sequential bargaining games, a formal form consists of the player set, the bargaining procedure, and the discount parameters. Although the mental level is much less obvious, this paper is not the place to actually propose a behavioral bargaining theory (see Güth, 1995). But one descriptively sensible explanation runs as follows. Players do not consider the consequences of their behavior as far as the diminishing value of any agreed upon price. Nor do they tune to the exact pattern of the negotiation – except for who makes the first offer, the restrictive nature of the ultimatum bargaining, and to one very egocentric aspect: the possible falling out of the negotiation altogether. Dropping out of the bargaining can happen while playing extended three-person games. These aspects are very salient and are picked up at the mental level.

Since player 1 is the first to actually make an offer, she enjoys a privileged position based on tradition and custom. Having one player who makes the offers is perceived as all too powerful with no natural justification – hence his limitation due to fairness, etc. Other patterns of bargaining, and in particular the strictly alternating bargaining embodied in the standard game, result in a mental symmetry. It pushes payoffs to reflect this fact which is in general correct, but its disregard of the discount rate is erroneous.

The most interesting aberration is caused, in our view, by the red herring provided by the extra buyer. This aspect is very clear but not completely thought out by the players, which is the reason to being characterized as egocentric. After all, why would a seller demand a higher unwarranted price if the alternative is playing against another, identical buyer (except for a label) one period later? [11] A myopic, egocentric thought could, on the other hand, consider the price setting (which is the value put on it by a buyer) as a

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[10] The reader might object because the means for 1/6 ultimatum show a reversed relation. Note, however, that the difference (0.3) between the means is the smallest one found and compared to the other difference for standard games it is only about 30%. We attribute it to random variation.

[11] In fact, as we have seen that sellers do switch buyers, hoping for a better outcome, more so in the practice plays than in cash ones. Of course, one has to take account of the fact that their initial demands were almost always accepted while playing cash games.

selling of a lottery. [12] One does not have to be a very astute decision maker under this assumption. Consider the standard game. It has a price – a certainty value. Now compare it to a modified game with the same parameters. Suppose that the seller and one of the buyers were already chosen and that the role of the other buyer to whom the first price offer is to be directed is auctioned out. The profits from playing a standard game are well known. As a bidder for this position (regardless of the auction type) you would pay less than the known profit in standard games because there is no guarantee that *you* will be the one to make a deal with the seller. You may even mentally construct the causal chain leading to your dismissal from the bargaining: "if I reject this offer the seller may punish me. What is there for him to lose?" This reasoning relies on ignoring losses incurred by a seller who would opt to change a buyer while considering a price offer. It belongs to the well-known genre where multiperson games are considered merely as one-person games played by each player (Camerer, 1990, Chapter 13).

Related to this description of the players' mental processes is an experiment reported by Weg and Zwick (1994), in which subjects bargained in a fixed-costs regime. In contrast to the standard version explored by Rapoport, Weg, and Felsenthal (1990) and by Weg and Zwick (1991), subjects were permitted to terminate the bargaining, settling any bargaining costs, with neither player gaining anything. Weg and Zwick found that, contrary to the subgame perfect prediction that opting out should have no effect on the bargaining outcome, subjects substantially mitigate their price demands (compared to their behavior in standard games). In this case, it was argued that empty game-theoretic threats are perceived as threatening indeed. In the present experiment, the increase in price demands (as compared to the decrease just mentioned) is explained by the ownership of the outside option. Here it is the seller who can change the monetary fate of the other players while in the fixed cost setup it is mainly the buyer (player 2) who holds the fate of the seller.

6. Discussion

Whereas Binmore et al. (1989, 1991), and Weg et al. (1996) may lead one to accept that behavior in discount two-person bargaining is fundamentally

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[12] In fact, an experiment on ultimatum bargaining where positions are auctioned by the second price method was conducted by Güth and Tietz (1986).

accounted for by subgame perfect rationality, experiments with finite discount rate games (e.g., Ochs & Roth, 1989), infinite horizon discount rate games (Weg et al., 1990), and the present experiment lead us to believe that in the context of normal experience accumulated in a lab, most subjects are sublimely ignorant of the recursive nature of *extensive* game bargaining. In fact, an important difference between the present experiment and those supporting subgame perfect rationality lies in the *realization* of the outside options. When some outside options are in fact *bargaining* options, as they are here, rather than fixed sums, their value is not transparent. It could be that the presence of certainty values as outside options combined with the *particular* choice of these concrete options have contributed to what can only be described as special cases of success of game-theoretic reasoning. But in general, subgame perfect equilibrium cannot be the corner stone of behavioral discount rate bargaining theory.

Experiments which address behavior in ultimatum scenario also employed three and even more participants in various arrangements (Roth et al., 1991; Güth et al., 1997; Güth & van Damme, 1998). *The correct* action in pure ultimatum was never contested. It is the overwhelming uniformity of rejection of rational choice displayed by participants the world over, that drove and continues to drive experimental work in ultimatum and its variations, in an effort to mark the boundaries of such a paradox. The ultimatum variations quoted here brought subjects into an easy conformity with rational choice. This is certainly not what is found in the present experiment. But what shows itself as promoting rational behavior in ultimatum, helping subjects to overcome the fairness barrier, merely helps promoting a more pronounced deviation from symmetry, which is not a barrier at all, but the ultimate proof of cluelessness. Thus, it does not in itself reflect a reasoned behavior but rather a consensual justification for extra greed.

This experiment, as well as many others in the bargaining domain (see Roth, 1995, for an extensive review), is modeled on versions of the Rubinstein (1982) seminal paper. While its importance to the advance of solution concepts of extensive games is not in doubt, its contribution to the understanding of naturally occurring conflict resolution scenarios is. One ought to bear in mind that Rubinstein's formalization of bargaining supports a more natural conceptualization than that of the finite horizon version hitherto known. [13] But this simplification is not sufficient. It is evident from the two

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[13] See Rubinstein (1991) for a related discussion.

limiting situations tested in the present experiment that subjects are utterly insensitive to the particular bargaining rule – sellers demand the same price regardless of whether they alternate offers with buyers (one extreme) or they themselves are the sole proposers (the other). Interpolation shows that any particular application for design and planning is not bound to achieve its purpose.

The main significance of the experiment is in the light it sheds on the more general field of markets where there are multitudes of buyers and sellers for the same or similar products. In cases where it is plausible to assume a prevailing and commonly known discount rate and where access time to new counterparts buyers or sellers is insignificant, one can break the market participants into two equivalent classes: the class comprised of all buyers and the class of all sellers. The market itself can be looked at as a game between two equivalent classes embodied as Rubinstein's original game. Our experiment suggests that we cannot clearly define a price, and if we do it is neither strategically determined nor robust. The necessity of revamping the notion of non-cooperative bargaining and its presumed foundation of markets in general (Osborne & Rubinstein 1990; Binmore, Osborne & Rubinstein, 1992,, Chapter 7) is clearly apparent, but the particulars of this undertaking are yet to be determined.

Acknowledgements

Appendix A. A bargaining study [14]

Instructions to subjects:
In this study, you can earn a substantial amount of money. Your earnings depend on your ability to negotiate a good deal for yourself.

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[14] This is a complete set of instructions. Bracketed prefixes signify specialization: 1 or 2 buyers and standard or ultimative game. Items without bracketed qualifications apply to all sessions. Same instructions to the two discount levels.

At the end of the session the supervisor will pay you discreetly so no one else will know how much money you made. And, of course, you will not know anybody else's gains.

Exploit this opportunity and try to gain as much money as you possibly can.

INTRODUCTION

In this session we simulate, in a limited way, the market for used textbooks.

[1s, 1u] The Seller

When we ask you to play the role of the Seller, assume that you have just completed MKTG 101. You have no intention to pursue this topic any further and you wish to sell your used textbook. THE MONEY YOU WILL GET FROM THE SALE IS YOURS TO KEEP.

Suppose that the bookstore stocked enough used copies of this textbook and is not looking for any more, and that only one student (described below as the Buyer) is interested in your used textbook. Thus, your only option is to sell your used textbook directly to this buyer at a price determined by a direct negotiation between the two of you.

The book by itself has no value to you other than its selling price.

[1s, 1u] The Buyer

When we ask you to play the role of a Buyer, assume that you are about to take MKTG 101. Your parents gave you $10 to buy the required textbook for this class. YOUR PARENTS AGREED THAT ANY MONEY LEFT OVER (IF YOU BUY THE BOOK FOR LESS) IS YOURS TO KEEP.

Suppose that the bookstore sells this used textbook for more than $10 (non-negotiable), and that only one student (described above as the Seller) is willing to sell you this book directly. Thus, your only option is to buy the used textbook directly from this student at a price determined by a direct negotiation between the two of you.

[1s, 1u] THE BARGAINING SESSION

During the session you will play the game nine times, six games for practice and three "for cash". In the practice games, you will play each role (SELLER, and BUYER) three times. However, you will play the same role in all three for cash games. Your role in these games will be assigned randomly after completion of the six practice games.

You will play each of the three for cash games with a different player; never the same player twice. Moreover, we have arranged the games such that your

co-bargainers in the ''for cash'' games are different from the ones you have encountered in the practice games.

You will not know your co-bargainers' identity, nor will they know yours. Nor will these identities be revealed after the session is completed.

[2s, 2u] The Seller

When we ask you to play the role of the Seller, assume that you have just completed MKTG 101. You have no intention to pursue this topic any further and you wish to sell your used textbook. THE MONEY YOU WILL GET FROM THE SALE IS YOURS TO KEEP.

Suppose that the bookstore stocked enough used copies of this textbook and is not looking for any more, and that only two students (described below as BUYER BLUE and BUYER YELLOW) are interested in your used textbook. Thus, your only option is to sell your used textbook directly to one of the Buyers at a price determined by a direct negotiation between the two of you.

The book by itself has no value to you other than its selling price.

[2s, 2u] The Buyers (BUYER BLUE and BUYER YELLOW)

When we ask you to play the role of a Buyer (BUYER BLUE or BUYER YELLOW), assume that you are about to take MKTG 101. Your parents gave you $10 to buy the required textbook for this class. YOUR PARENTS AGREED THAT ANY MONEY LEFT OVER (IF YOU BUY THE BOOK FOR LESS) IS YOURS TO KEEP.

Suppose that the bookstore sells this used textbook for $10 (non-negotiable), and that only one student (described above as the Seller) is willing to sell you this book directly. Thus, you can either buy the used textbook directly from this student at a price determined by a direct negotiation between the two of you, or buy it from the bookstore at a fixed price of $10.

The used textbook offered by the Seller, and the used textbook offered by the bookstore are identical for all practical purposes.

[2s, 2u] THE BARGAINING SESSION

During the session you will play the game nine times, six games for practice and three ''for cash''. In the practice games you will play each role (SELLER, BUYER BLUE, and BUYER YELLOW) twice. However, you will play the same role in all three ''for cash'' games. Your role in these games will be assigned randomly after completion of the six practice games.

You will play each of the three ''for cash'' games with different players; never the same players twice. Moreover, we have arranged the games such

that your co-bargainers in the "for cash" games are different from the ones you have encountered in the practice games.

You will not know your co-bargainers' identity, nor will they know yours. Nor will these identities be revealed after the session is completed.

[1s] How do you bargain?

The game takes place in rounds. On each round, one of the players makes a price offer, and the other player can either accept or reject this offer. After rejection, the game moves to the next round.

The Seller starts the game by making a price offer to the Buyer. The Buyer has two options:
1. Accept the price offer. Here, the Seller sells the book to the Buyer at the agreed price.
2. Reject the price offer. This choice signals an intent to continue the bargaining and it is the Buyer's turn to propose a price in the next round.
After a Buyer makes an offer, the Seller has two options:
1. Accept the price offer. Here, the Seller sells the book to the Buyer at the agreed price.
2. Reject the price offer. This choice signals an intent to continue the bargaining and it is the Seller's turn to propose a price in the next round, and the process repeats itself.

[1u] How do you bargain?

The game takes place in rounds. On each round, the Seller makes a price offer to the Buyer. The Buyer has two options:
1. Accept the price offer. Here, the Seller sells the book to the Buyer at the agreed price.
2. Reject the price offer. After rejection, the Seller makes a new price offer in the next round, and the process repeat itself.

[2s] How do you bargain?

The game takes place in rounds. On each round, one of the players makes a price offer, and the player to whom the offer is made can either accept or reject this offer. After rejection, the game moves to the next round.

The Seller starts the game by making a price offer to one of the Buyers. It is entirely up to the Seller to decide to whom to make the offer. The Buyer to whom the offer is made has two options:
1. Accept the price offer. Here, the Seller sells the book to that Buyer at the agreed price. The other Buyer is forced to buy the book from the bookstore for $10.

2. Reject the price offer. This choice signals an intent to continue the bargaining and it is the Buyer's turn to propose a price in the next round.

After a Buyer makes an offer, the Seller has two options:

1. Accept the price offer. Here, the Seller sells the book to that Buyer at the agreed price. The other Buyer is forced to buy the book from the bookstore for $10.
2. Reject the price offer. After rejection, the Seller decides to whom to make a price offer in the next round, and the process repeats itself.

[2u] How do you bargain?

The game takes place in rounds. On each round, the Seller makes a price offer to one of the Buyers. It is entirely up to the Seller to decide to whom to make the offer. The Buyer to whom the offer is made has two options:

1. Accept the price offer. Here, the Seller sells the book to that Buyer at the agreed price. The other Buyer is forced to buy the book from the bookstore for $10.
2. Reject the price offer. After rejection, the Seller decides to whom to make a price offer in the next round, and the process repeats itself.

[2u] What is your profit in a game?

If you are the Seller, your profit is the money you make by selling the book. If you are the Buyer, your profit is whatever is left of the $10 you got from your parents after paying for the book.

Time is Money. As in real life bargaining is costly. The costs are due to the time it takes to negotiate a deal. As you well know "time is money".

What are the costs involved in bargaining?

The present study incorporates delay costs by paying the players only some known percentage of their profits conditional on the time of the agreement – this is their net profits.

Specifically, in this study bargainers will receive 100% of their profits if agreement is reached in round 1, 16.67% if agreement is reached in round 2, 2.78% if agreement is reached in round 3, 0.46% if agreement is reached in round 4, and so on. You can actually compute these values by using the formula:

$$100 \times (1/6)^{(\text{round of agreement}-1)}.$$

You do not need to calculate these values. Rather, these values, referred to as "percentage of profit paid," will be presented to you on the computer screen.

Furthermore, on each round, given a price offer, the computer will display the net profits, taking into account the costs of delay.

How do you get paid for your participation?

We shall pay you $5.00 as a show up fee. In addition, at the end of the experiment, we shall pay you your total (accumulated) net profits in the three "for cash" games.

At the end of the session the supervisor will pay you discreetly so that no one else will know how much money you have made. And, of course, you will not be informed of the payoffs of the other participants.

How to use the computer terminal?

Please use the computer to communicate with your co-bargainers. Use the keyboard to write numbers and signal agreements or rejections. To write the amounts, use the digit keys. The decimal point is already marked on the screen; therefore, if a sum is less than $10 enter the digit "0" first. End each entry by pressing the "Enter" key. If you want to erase a character, use the "Backspace" key.

Consent Form. If you wish to participate in this study, then please read and sign the accompanying consent form.

Note. In order to collect your earnings you must stay until the end of the session, which will last no more than two hours. From now until the end of the session, unauthorized communication of any nature with other participants is prohibited.

If you have questions, press the "H" key now and wait until the supervisor arrives. You can use the "H" key to obtain help from the supervisor at any time during the session.

If you have understood these instructions, and are ready to start the practice games, press the "G" key now.

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