Coherence protection for quantum computation

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Why quantum computation?

Richard Feynman: if simulating quantum mechanical objects is so difficult, we ought to be able to use this for computation

Example: classical three-bit register can be in one of 8 states: (000), (001), (010), (011), (100), (101), (110),(111).

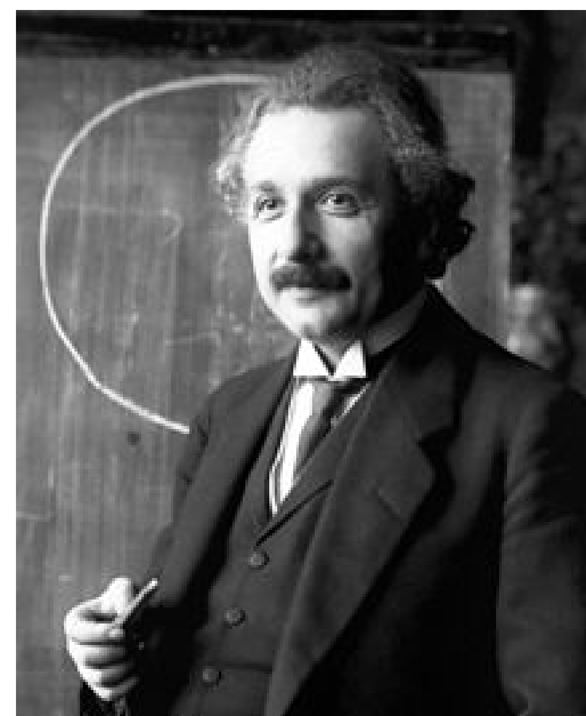
A register of three quantum bits (qbits) can be in a superposition of all of these states:
$|\psi\rangle = A_0\,|000\rangle + A_1\,|001\rangle + A_2\,|010\rangle + A_3\,|011\rangle + A_4\,|100\rangle + A_5\,|101\rangle + A_6\,|110\rangle + A_7\,|111\rangle$

Measurement: if a single qubit is in a superposition state, $A_0\,|0\rangle + A_1\,|1\rangle$, measurement in this basis would return 0 with probability $|A_0|^2$ (leaving the qubit in the state $|0\rangle$), or 1 with probability $|A_1|^2$ (leaving the qubit in the state $|1\rangle$).

Quantum entanglement

Example: put two qubits in the "cat" state $|00\rangle/\sqrt{2} + |11\rangle/\sqrt{2}$. Now, even if the qubits are far apart, when we measure onequbit, with equal probabilities, the wavefunction gets projected to either of the states $|00\rangle$ and $|11\rangle$. Subsequent measurement of the other qubit always returns the same value, even if the qubits are far separated.

In special relativity, simultaneity of two events in different locations is not absolute: even when events are simultaneous in one reference frame, there are reference frames where one or the other event occurs earlier.

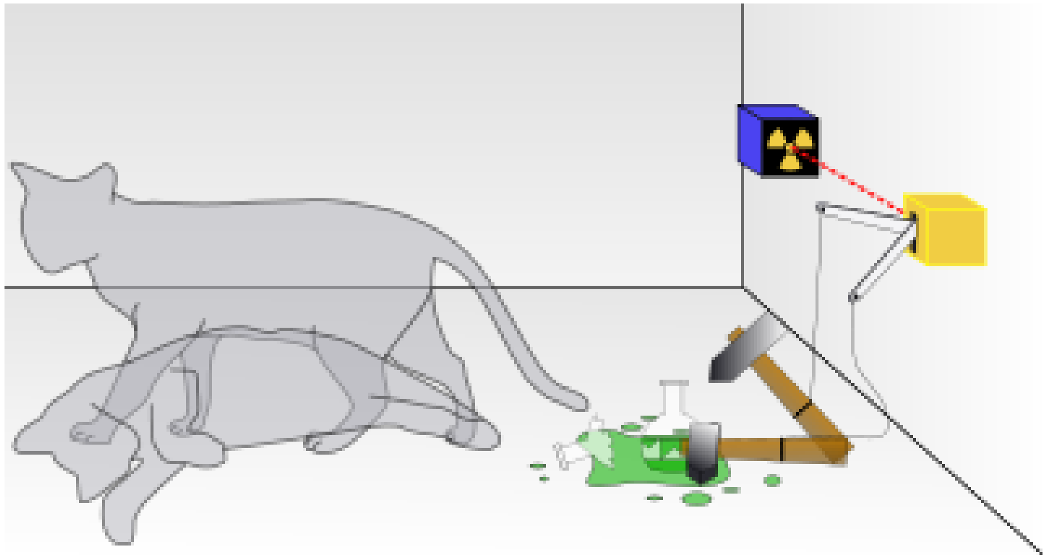


Albert Einstein famously derided entanglement as "spukhafte Fernwirkung" or "spooky action at a distance".

In reality, no violation of special relativity occurs, as there is no way entanglement can be used to transmit information faster than light.

Schrödinger's cat

A cat, a flask of poison and a radioactive source are placed in a sealed box. If an internal Geiger counter detects radiation, the flask is shattered, releasing the poison that kills the cat. The Copenhagen interpretation of quantum mechanics implies that after a while, the cat is simultaneously alive and dead. Yet, when we look in the box, we see the cat either alive or dead, not both alive and dead.



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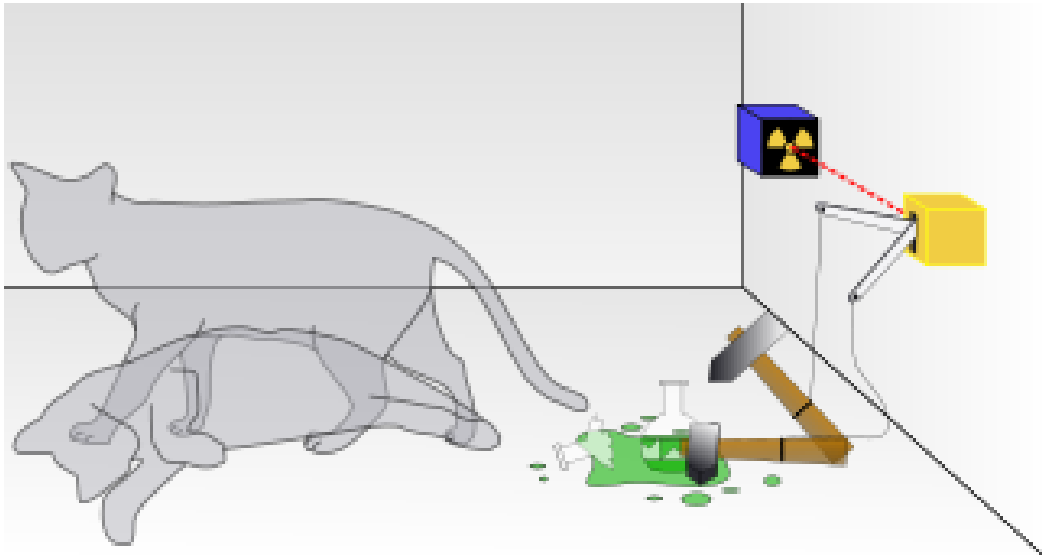
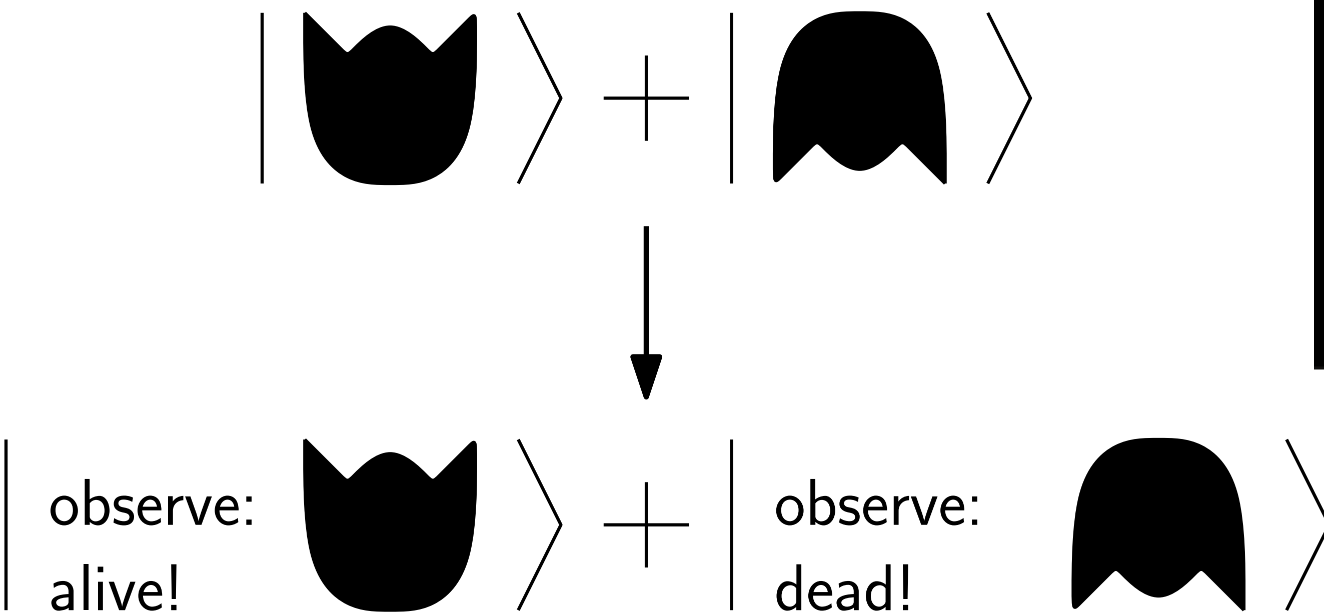


In the macroscopic world, it is hard to observe quantum-mechanical entanglement due to decoherence.

Decoherence

Environment constantly gets entangled with the quantum system



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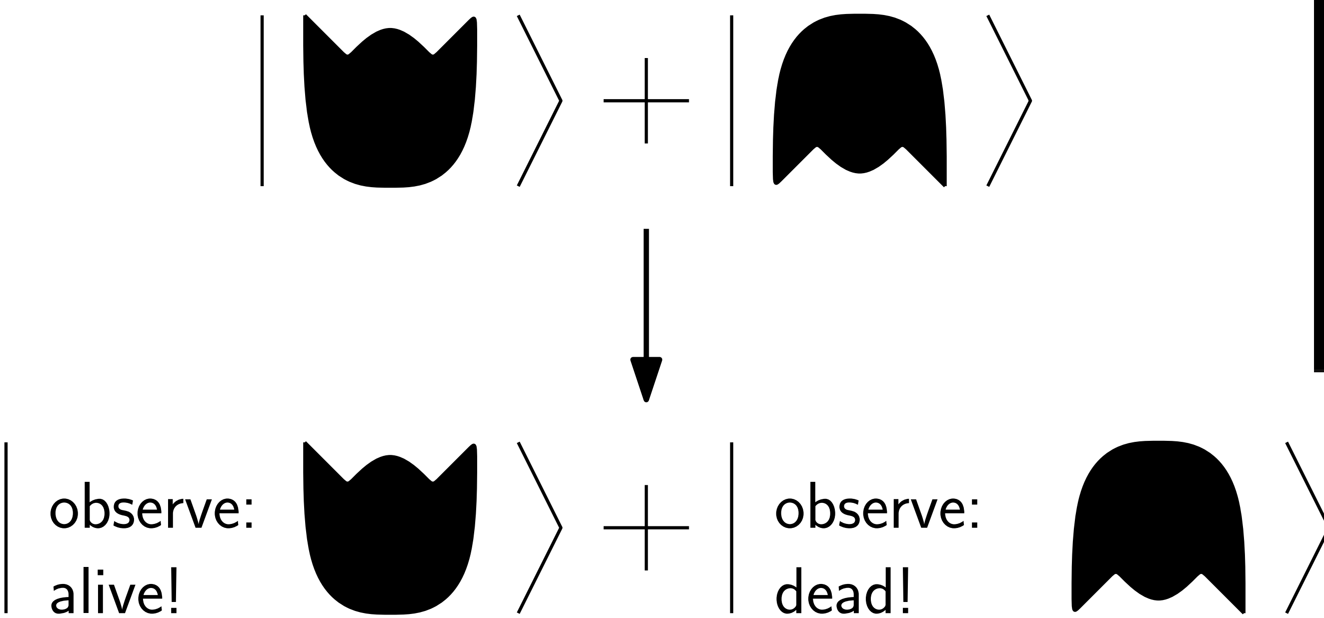

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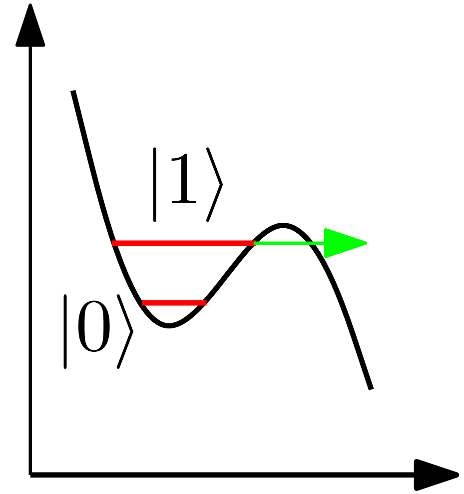
Preventing decoherence is the major challenge in designing a quantum computer

Continuous quantum measurement example

Null measurement for superconducting qubit

- Prepare the wavefunction $\psi_0 = \alpha\,|0\rangle + \beta\,|1\rangle$
- Allow tunneling for time t: $\psi \to A(\alpha\,|0\rangle + e^{i\phi}\sqrt{p}\beta\,|1\rangle)$, where p is the tunneling probability
- Surprise: state evolves non-trivially, but remains pure



John Martinis

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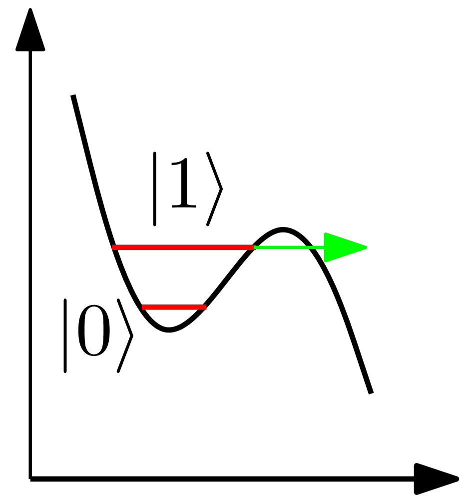

Classical analogy: perceived probability evolution for a coin in a dark room

$P_{\text{tails}} = \frac{1}{2}(1+p)$, $P_{\text{heads}} = \frac{1}{2}(1-p)$

Decoherence

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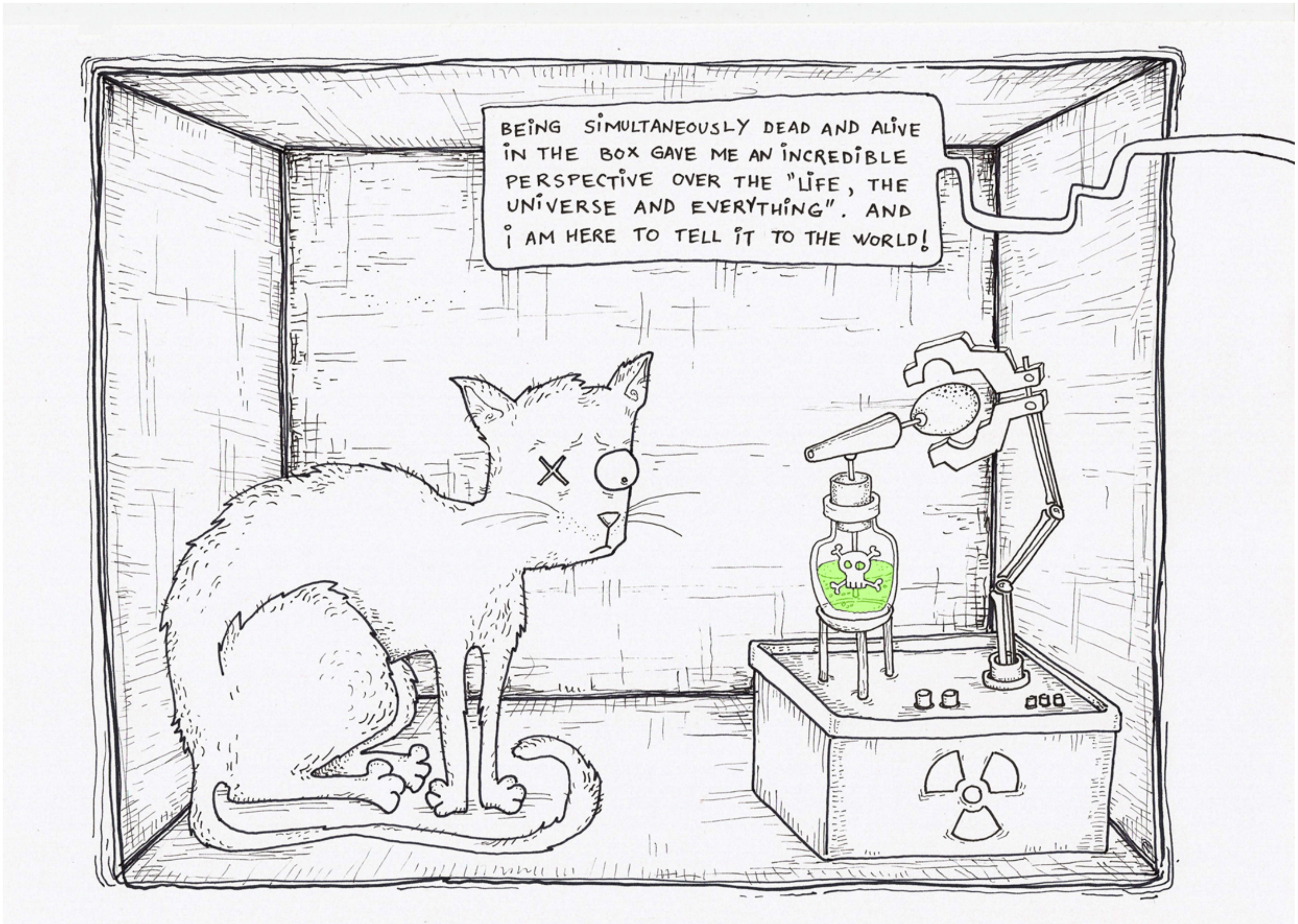

Classical error correction

Classical example: redundancy code

Information bits: 0 or 1

Transmitted bits: (000) or (111)

Code distance: $d = 3$. You can detect any two bit-flip errors, and correct any two such errors

Error correction protocol: majority vote

You can have longer redundancy codes, e.g., 5-bit, 7-bit, ... to correct $t = 2, 3, \ldots$ errors.

Quantum error correction

Invented by Peter Shor

Opened up quantum computation as a theoretical possibility

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Main hurdle: how can you introduce redundancy if you are not allowed to measure the individual qubits (measurement destroys quantum state)

Example: three-qubit repetition code

Code space formed by linear combinations of the basis vectors

$$|\bar{0}\rangle \equiv |000\rangle\,, |\bar{1}\rangle \equiv |111\rangle$$

Logical operators

$$\overline{X} = XXX, \quad \overline{Z} = ZZZ, \quad \overline{Y} = i\overline{XZ} = -YYY$$

Check for bit-flip errors by measuring the operators

$$S_1 = ZZI, \quad S_2 = IZZ$$

$$\begin{aligned}
|\psi\rangle = A|\bar{0}\rangle + B|\bar{1}\rangle \quad &\Rightarrow \quad \langle S_1\rangle = 1,\ \langle S_2\rangle = 1 \\
\left|\tilde{\psi}\right\rangle = X_1|\psi\rangle \quad &\Rightarrow \quad \langle S_1\rangle = -1,\ \langle S_2\rangle = 1 \\
\left|\tilde{\psi}\right\rangle = X_2|\psi\rangle \quad &\Rightarrow \quad \langle S_1\rangle = -1,\ \langle S_2\rangle = -1 \\
\left|\tilde{\psi}\right\rangle = X_3|\psi\rangle \quad &\Rightarrow \quad \langle S_1\rangle = 1,\ \langle S_2\rangle = -1
\end{aligned}$$

Resolution: measure a syndrome based on a set of commuting operators which do not address individual qubits.

General stabilizer codes

Code $\mathcal{Q}$ is stabilized by the Abelian *stabilizer group* $\mathscr{S} \subset \mathscr{P}_n$

$$\mathcal{Q} \equiv \{ |\psi\rangle : S\,|\psi\rangle = |\psi\rangle\,, \forall S \in \mathscr{S} \}$$

Such a code exists as long as $-\mathbf{1} \notin \mathscr{S}$.
If $\mathscr{S} = \langle S_1, \ldots, S_{n-k} \rangle$, with $(n-k)$ generators, the code encodes k logical qubits.
There are k logical operators $\overline{X}_i$, $\overline{Z}_i$, $i = 1, \ldots, k$ which commute with every element in $\mathscr{S}$. The code is denoted $[[n,k,d]]$, where d is the distance of the code.

Daniel Gottesman

The group $\langle S_1, \ldots, S_{n-k}, Z_1, \ldots, Z_k \rangle$ stabilizes a unique stabilizer state $|s\rangle \equiv |\bar{0} \ldots \bar{0}\rangle$; the basis of the code is $|\alpha_1, \ldots \alpha_k\rangle \equiv \overline{X}_1^{\alpha_1} \ldots \overline{X}_k^{\alpha_k}\,|s\rangle$, $\alpha_j = \{0,1\}$, $j = 1, \ldots, k$.

Errors are detected by measuring the generators S_i of the stabilizer $\mathscr{S}$ (binary syndrome vector)

Example: [[5,1,3]] stabilizer code

$\mathcal{Q} \equiv \{|\psi\rangle : G_i |\psi\rangle = |\psi\rangle, i = 1, \ldots, 4\}$ with generators
$S_1 = XZZXI$, $S_2 = IXZZX$, $S_3 = XIXZZ$, $S_4 = ZXIXZ$
A basis of the code space is (up to normalization)

$$|\bar{0}\rangle = \prod_{i=1}^{4}(\mathbf{1} + G_i)|00000\rangle\,, \quad |\bar{1}\rangle = \overline{X}\,|\bar{0}\rangle\,.$$

The logical operators can be taken as

$$\overline{X} = ZZZZZ, \quad \overline{Z} = XXXXX.$$

Measure generators of the stabilizer to find the error, e.g.,
$\left|\tilde{\psi}\right\rangle = X_1(A|\bar{0}\rangle + B|\bar{1}\rangle)$ gives unique syndrome

$$\langle S_1\rangle = 1,\ \langle S_2\rangle = 1,\ \langle S_3\rangle = 1,\ \langle S_4\rangle = -1.$$

For this code, there are total of 15 single-qubit errors, and exactly 15 distinct syndromes (apart from $\langle S_i\rangle = 1$ for any $|\psi\rangle \in \mathcal{Q}$).

Quantum threshold theorem

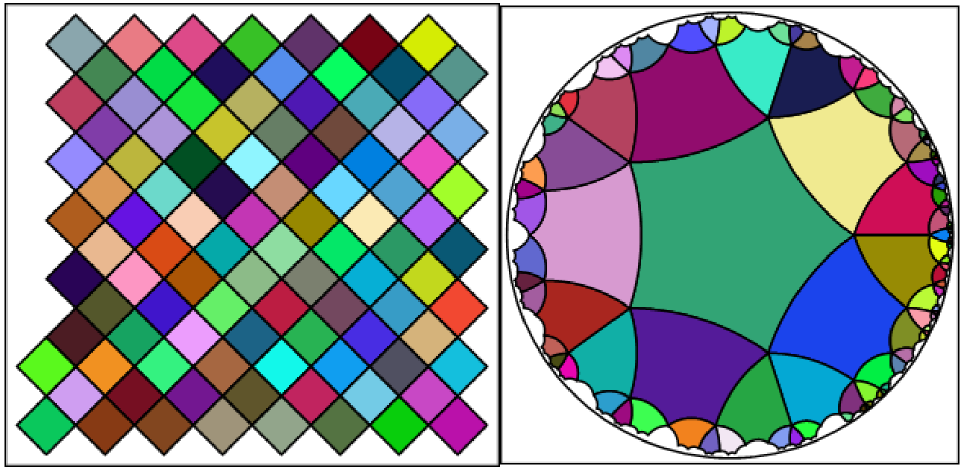
If the error probability per elementary operation is small enough, one can design an error correction scheme to sustain an arbitrary large quantum computation

Original version relied on multiple layers of encoding and resulted in a threshold error probability of $\sim 10^{-4}$

Present value of the threshold is around a percent, based on so-called surface codes

Still, huge overhead in the number of necessary auxiliary qubits

Surface quantum codes: each qubit corresponds to an edge of a planar graph



My research

Try to design new code families which would allow for higher threshold value with a smaller overhead

Try to come up with quantum control protocols where some of the errors are suppressed before they happened — thereby reducing the requirements for quantum error correcting code

Try to put these two together in a simulation in order to provide clear instructions for experimentalists