Progressive Taxation and Long-Run Income Inequality under Endogenous Growth*

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Abstract

This paper examines the theoretical and quantitative interrelations between progressive taxation and after-tax income inequality within a one-sector endogenous growth model. In a simplified two-type version of the macroeconomy, we analytically show that higher fiscal progression leads to less unequal long-run gross/net income distributions, provided the general-equilibrium elasticity of aggregate labor supply surpasses a specific negative threshold. In addition, numerical simulations find that our calibrated economy under useless or useful government spending, together with (i) agents' intertemporal elasticity of consumption substitution taking on the highest empirically-plausible value and (ii) a lower-than-unitary elasticity of capital-labor substitution in production, is able to generate qualitatively as well as quantitatively realistic long-run disposable-income inequality effects of changing the tax progressivity *vis-à-vis* recent panel estimation results.

Keywords: Progressive Taxation; Income Inequality; Endogenous Growth.

JEL Classification: D31; E13; H30.

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1 Introduction

Under the progressive fiscal policy rule, wealthy individuals face a higher tax rate on their factor income and pay a larger share of the tax burden. This budgetary scheme in turn tends to mitigate the economy's after-tax income inequality. Based on a panel dataset from 165 countries over the 1981 – 2005 period, Ducan and Sabirianova Peter (2016) report that a one-unit increase in the average rate progression (ARP)[1] reduces the value of the disposable-income Gini coefficient/index by 1.194 per OLS regression; 4.069 per unweighted instrumental variable estimation; and 2.61 per the weighted instrumental variable approach. As a result, the calculated elasticities of post-tax Gini with respect to the average rate progression (in absolute terms) range over the interval [0.1206, 0.411]. The objective of this paper is to develop a dynamic general equilibrium model that is able to yield qualitatively as well as quantitatively realistic long-run net-income-inequality effects of progressive taxation in accordance with the aforementioned sizable and statistically significant (at the 1% or 5% level) empirical findings.[2] Our work is thus a piece of positive macroeconomics research which abstracts from deriving the optimal fiscal policy and/or examining the associated normative/welfare issues.

We begin with modifying the one-sector endogenous growth model, analyzed by Koyuncu and Turnovsky (2016), that features heterogeneous infinitely-lived households, variable labor supply and a progressive tax schedule *à la* the spirit of Guo and Lansing (1998) in continuous time. Agents derive utilities from consumption and leisure through a non-separable and isoelastic preference formulation. They differ in terms of initial capital endowments and time preference rates, with the latter generating non-degenerate dispersions in key macroeconomic variables since progressive income taxation entails differentiated after-tax returns to their capital investment. Motivated by recent empirical evidence of Knoblach et al. (2020) and Gechert et al. (2022) which casts doubt on the conventional Cobb-Douglas technological specification as in the Koyuncu-Turnovsky framework, a constant elasticity-of-substitution (CES) production technology with positive productive externalities from aggregate capital stock is adopted here.[3] The government balances the budget at each instant of time by spending its tax revenue on goods and services that are postulated to be useless or wasteful within our baseline setting.

[1] ARP characterizes the structural progressivity of a tax schedule in terms of the changes in average tax rates along the income distribution. Its mathematical expression is given by $ARP = \frac{\tau^m - \overline{\tau}}{\overline{\tau}}$, where τ^m and τ denote the marginal and average tax rates, respectively.

[2] Using multiple sample panels from 24 and 28 OECD countries for the period of 1981 – 2005, Doerrenberg and Peichl (2014) find that the effects of a one-percent increase in the tax progressivity on post-tax income inequality are small (between −0.04% and −0.2%) and often statistically insignificant.

[3] See García-Peñalosa and Turnovsky (2009) for a theoretical discussion on how production flexibility affects the capital/wealth distributional inequality in a Ramsey model with heterogeneous agents and fixed labor supply.

Our analysis is focused on the model's unique long-run balanced growth path (BGP) along which individual as well as aggregate labor hours, together with relative shares of capital/wealth and income across households are stationary; whereas the economy-wide levels of output, consumption and physical capital all grow at a common positive constant rate. In a simplified two-type version of the macroeconomy, we analytically demonstrate that an increase in the fiscal progression leads to less unequal distributions of gross/disposable income in the long run and a lower growth rate of GDP, provided the general-equilibrium elasticity of aggregate labor supply with respect to the degree of tax progressivity surpasses a specific negative threshold (a sufficient condition). Intuitively, more progressive income taxation immediately raises the real wage while reducing the interest rate because of a decline in the economy-wide level of hours worked. In addition, the average tax rate faced by impatient households falls on impact, whereas that for patient individuals rises. At the economy's new steady-state equilibrium, capital-poor agents work harder and own a larger share of national wealth than their initial allocations. This in turn leads to reductions in before- and after-tax income inequality. We also show that if the above-mentioned requisite condition is satisfied, higher fiscal progression will decrease the economy's output growth rate due to a smaller net rate of return on capital investment.

Since the dynamic interrelations among BGP quantities in our endogenously growing macroeconomy with more than three differentiated groups of households are not analytically tractable, a quantitative assessment on the long-run distributional/growth effects of more progressive income taxation is undertaken. In contrast to Koyuncu and Turnovsky's (2016) two-type numerical experiments, we examine five heterogeneous groups of individuals with a monotonically decreasing sequence of time preference rates such that the model's beginning stationary quintile distribution of after-tax income shares matches with the panel data from a sample of 32 OECD countries over the $2008 - 2017$ period. When the tax-progressivity parameter rises within our baseline model, the bottom three quintiles of agents are found to increase their hours worked on impact, and then work less during the transition instants of time. These capital-poor individuals will also raise their investment expenditures immediately because of higher post-tax capital rental rates, and then slow down their capital accumulation rate along the transition path. On the other hand, labor supply and savings responses of the highest two quintiles of capital-rich households are of exactly the opposite directions. As a result, the new steady-state distributions of agents' relative capital/wealth shares and labor hours, as well as pre-tax and after-tax income, are less dispersed than their original counterparts. Under our benchmark parameterization, a one-percentage-point increase in the average rate progression generates a decrease in the post-tax Gini coefficient by 1.92%, which in turn leads to a calculated elasticity (in absolute value) of 0.0693.[4] This figure is substantially lower than the estimated range of $0.1206 - 0.411$ reported by Ducan and Sabirianova Peter (2016).

In terms of sensitivity analysis, we consider alternative empirically-plausible calibrations on agents' intertemporal elasticity of substitution (IES) in consumption and firms' elasticity of substitution (ELS) between capital and labor inputs in production, while re-calibrating other parameters to achieve that each parametric configuration starts at the same balanced-growth equilibrium as in the baseline framework. As it turns out, a higher IES enables households to more readily adjust their consumption spending across different instants of time and generates a larger reduction in the interest rate on impact of more progressive taxation. This effect will speed up (slow down) capital-poor (capital-rich) agents' wealth accumulation rate during the subsequent transition, resulting in a less unequal long-run disposable-income distribution. However, the calculated elasticities are still unrealistically low for our benchmark parameterization with $ELS = 0.87$, a non-negligible departure from the Cobb-Douglas specification *à la* Koyuncu and Turnovsky (2016). Our numerical experiments additionally find that under higher fiscal progression, whether a change in the elasticity of factor substitution amplifies or mitigates the decline in after-tax income inequality depends on the relative strength of quantitative impacts on capital/wealth dispersion, labor supply, and factor income shares. When agents' intertemporal elasticity of consumption substitution is equal to one (the separable log-log utility function) or two, the Gini coefficient on net income is *ceteris paribus* monotonically increasing with respect to the capital-labor substitution elasticity; and that the reverse holds true when $IES = \frac{1}{3}$. In particular, the calibrated macroeconomy with $ELS = 0.45$ and $IES = 2$ delivers a calculated elasticity ($= 0.1404$) that is larger than the lower bound of the estimated interval [0.1206, 0.411].

Next, we examine an otherwise identical one-sector endogenous growth model with useful government purchases of goods and services, either being productivity-enhancing or utility-generating. Under productive public spending *à la* Chatterjee and Turnovsky (2012), more progressive taxation leads to larger decreases in the aggregate level of hours worked and the rate of return to capital investment on impact. When the household utility function is logarithmically separable or exhibits less risk aversion, these immediate effects will prompt capital-poor individuals to reduce their future labor supply at a faster rate as well as yield a greater decline in the steady-state capital share of national income, both of which result in a less unequal

[4] For the sake of analytical robustness, our model economy without elastic hours worked and/or sustained long-run economic growth is also examined. For the compatible baseline calibration of each variant under consideration, we find that the calculated elasticity is lower than 0.0693 reported here in an endogenously growing macroeconomy with variable labor supply. These numerical results are available upon request from the authors.

distribution of disposable income in the long run. Specifically, the parametric combinations of $ELS = 0.87$ and $IES = 2$ generate a calculated elasticity ($= 0.12777$) that is a close match with the empirical evidence. We also find that under non-separable utility-generating public expenditures *à la* García-Peñalosa and Turnovsky (2011), the long-run distribution of agents' after-tax income will become less dispersed in response to higher fiscal progression when private consumption and public goods are Edgeworth complements with $IES = 2$. In this environment, lower government spending causes capital-poor individuals to speed up the decreases in their labor hours and capital accumulation along the transition path. Together with the baseline calibration of $ELS = 0.87$, our model's calculated elasticity is raised to 0.1915 which is significantly larger than the lower bound of the empirically-plausible range.

Overall, this paper shows that our calibrated one-sector endogenous growth model under useless or useful government spending, along with (i) agents' intertemporal elasticity of consumption substitution taking on the highest possible value that is regarded as empirically plausible ($IES = 2$) and (ii) a lower-than-unitary elasticity of substitution between capital and labor inputs in production, is able to generate qualitatively as well as quantitatively realistic long-run net-income-inequality effects of changing fiscal progression *vis-à-vis* recent panel estimation results from Ducan and Sabirianova Peter (2016). In terms of relevant references, Li and Sarte (2004) and Chen (2020) study the growth-inequality trade-off under progressive income taxation in an endogenously growing macroeconomy parameterized to the implementation of the U.S. Tax Reform Act of 1986; and Carneiro et al. (2022) examine a similar research topic in connection with the U.S. Tax Cuts and Jobs Act enacted in 2017. As in Koyuncu and Turnovsky (2016), these previous studies all adopt the conventional Cobb-Douglas production technology in their analyses. In addition, Li and Sarte's (2004) model considers inelastic labor supply; whereas Carneiro et al. (2022) postulate that individuals are differentiated by distinct intertemporal elasticities of substitution in consumption; and the numerical simulations of Koyuncu and Turnovsky (2016) and Chen (2020) also focus on transitional dynamics with two groups of heterogeneous households.

The remainder of this paper is organized as follows. Section 2 describes our baseline one-sector endogenous growth model with useless government spending under progressive income taxation, and discusses its equilibrium conditions. Section 3 derives the economy's unique balanced-growth path and the Gini coefficient associated with the long-run distribution of agents' disposable income. Section 4 quantitatively examines the net-income-inequality effects of higher fiscal progression within a calibrated version of our benchmark model. Section 5 studies an otherwise identical endogenously growing macroeconomy with productive or utility-generating public expenditures of final goods and services. Section 6 concludes.

2 The Model

Our analysis begins with incorporating a constant elasticity-of-substitution (CES) production function into Koyuncu and Turnovsky's (2016) one-sector endogenous growth model with heterogeneous infinitely-lived households, variable labor supply and progressive income taxation in continuous time. Agents derive utilities from consumption and leisure under a non-separable and isoelastic preference formulation, along with distinct rates of time preference and initial capital endowments. These heterogeneities in turn will affect each individual's attitude toward savings as well as propensity to work. The economy's production side consists of a social technology that displays increasing returns-to-scale due to positive productive externalities from the aggregate capital stock. The government balances the budget at each instant of time by spending its tax revenue on goods and services that do not contribute to the households' utility or the firms' production. We assume that there are no fundamental uncertainties present in this macroeconomy.

2.1 Firms

There is a continuum of identical competitive firms, with the total number normalized to one. The representative firm j produces output Y_{jt} according to a standard CES production function

$$Y_{jt} = A\left[\alpha K_{jt}^{\varepsilon} + (1-\alpha)(X_t H_{jt})^{\varepsilon}\right]^{\frac{1}{\varepsilon}}, \; A > 0, \quad 0 < \alpha < 1 \;\text{ and }\; -\infty < \varepsilon \leqslant 1, \tag{1}$$

where A is a technological scalar, K_{jt} and H_{jt} are capital and labor inputs, and the elasticity of substitution between productive factors is given by $ELS = \frac{1}{1-\varepsilon}$. When $\varepsilon \to 1$, labor and capital become perfect substitutes; when $\varepsilon \to -\infty$, the production technology takes a Leontief formulation such that capital and labor become perfect compliments; and when $\varepsilon \to 0$, we recover the conventional Cobb-Douglas production function as in Koyuncu and Turnovsky (2016).

In addition, X_t represents positive productive externalities that are taken as given by each individual firm, and it is postulated to take the form

$$X_t = K_t. \tag{2}$$

where $K_t \left(= \int_0^1 K_{jt} dj\right)$ denotes the economy's average/aggregate level of capital services. In a symmetric equilibrium, all firms make the same decisions such that $K_{jt} = K_t$ and $H_{jt} = H_t$ $\left(= \int_0^1 H_{jt} dj\right)$ for all j and t. As a result, we obtain an aggregate increasing returns-to-scale production technology for total output Y_t:

$$Y_t = A[\alpha + (1-\alpha)H_t^{\varepsilon}]^{\frac{1}{\varepsilon}} K_t. \tag{3}$$

It follows that our macroeconomy exhibits sustained long-run economic growth because the social technology (3) displays linearity in physical capital. Under the assumption that factor markets are perfectly competitive, the first-order conditions for the representative firm's profit maximization problem are

$$r_t = \alpha A[\alpha + (1-\alpha)H_t^{\varepsilon}]^{\frac{1-\varepsilon}{\varepsilon}}, \tag{4}$$

$$w_t = \underbrace{(1-\alpha)A[\alpha + (1-\alpha)H_t^{\varepsilon}]^{\frac{1-\varepsilon}{\varepsilon}} H_t^{\varepsilon-1}}_{\equiv\, \omega(H_t)} K_t, \tag{5}$$

where r_t is the capital rental rate and w_t is the real wage rate.

2.2 Households

The economy is inhabited by heterogeneous households that are indexed by $i = \{1, 2, ... M\}$. Individual i is endowed with one unit of labor hour at each instant of time; together with an initial level of capital stock $K_{i0} > 0$ and a subjective rate of time preference $\beta_i \in (0, 1)$. The objective for each agent is to maximize a discounted stream of utilities over its lifetime:

$$\int_0^\infty \underbrace{\frac{1}{\gamma}\left(C_{it}\ell_{it}^{\theta}\right)^{\gamma}}_{\equiv\, U_{it}} e^{-\beta_i t} dt, \; -\infty < \gamma < 1, \quad \theta > 0, \text{ and } \; \gamma\theta < 1, \tag{6}$$

where C_{it} is consumption, ℓ_{it} is leisure, the intertemporal elasticity of substitution in "effective consumption" $C_{it}\ell_{it}^{\theta}$ is given by $IES = \frac{1}{1-\gamma}$, and U_{it} is a homogenous utility function of degree $\gamma(1+\theta)$. Notice that when $\gamma = 0$, each household's preference formulation becomes separable and logarithmic in both consumption and leisure, *i.e.* $U_{it} = \log C_{it} + \theta \log \ell_{it}$.

The budget constraint faced by individual i is given by

$$\dot{K}_{it} = (1-\tau_{it}) \underbrace{(r_t K_{it} + w_t H_{it})}_{\equiv\, Y_{it}} - C_{it} - \delta K_{it}, \quad 0 < \tau_{it}, \; \delta < 1, \tag{7}$$

where H_{it} $(= 1 - \ell_{it})$ is hours worked, Y_{it} is household i's total factor income, τ_{it} is the income tax rate and δ the capital depreciation rate. As in Li and Sarte (2004) and Koyuncu and Turnovsky (2016), we postulate that the fiscal policy rule is continuously progressive per the spirit of Guo and Lansing (1998), with the income tax rate τ_{it} being specified as

$$\tau_{it} = \eta y_{it}^{\phi}, \tag{8}$$

where $y_{it} \left(\equiv \frac{Y_{it}}{\overline{Y}_t}\right)$ is the ratio of agent i's factor income relative to the economy-wide average level $\overline{Y}_t = r_t K_t + w_t H_t$, and the parameters η and ϕ govern the level and slope (or elasticity) of the tax schedule, respectively. We also postulate that agents are able to rationally anticipate the way in which changes to their income affect their tax burden. As a result, each household's economic decisions are governed by its individual marginal tax rate given by.

$$\tau_{it}^{m} = \frac{\partial(\tau_{it} Y_{it})}{\partial Y_{it}} = \eta(1+\phi) y_{it}^{\phi}. \tag{9}$$

Our analysis below focuses on an environment in which (i) agents have an incentive to provide capital and labor services to firms and (ii) the government cannot confiscate productive resources, thus $0 < \tau_{it},\ \tau_{it}^{m} < 1$ is imposed. At the model's symmetric equilibrium with $Y_{it} = \overline{Y}_t$ for all i and t, these conditions imply that $\eta \in (0,\ 1)$ and $\phi \in (-1,\ \frac{1-\eta}{\eta})$, where $\frac{1-\eta}{\eta} > 0$. It follows that when $\phi > (<)\ 0$, the tax scheme is said to be progressive (regressive), *i.e.* the marginal tax rate is higher (lower) than the corresponding average tax rate given by (8). When $\phi = 0$, the average and marginal tax rates coincide at the constant level of η, thus the tax schedule is flat. Consequently, the degree of tax progressivity associated with (8) is determined by the elasticity parameter ϕ. According to the observed progressive U.S. federal individual income tax schedule, the listed statutory marginal tax rate τ_{it}^{m} is an increasing and concave function with respect to the household's taxable-income ($= Y_{it}$) brackets. Therefore, the tax-progressivity parameter is further restricted to the interval $0 < \phi < \min\{1,\ \frac{1-\eta}{\eta}\}$.

The first-order conditions for household i's dynamic optimization problem are

$$C_{it}^{\gamma-1} \ell_{it}^{\gamma\theta} = \lambda_{it}, \tag{10}$$

$$\theta \frac{C_{it}}{\ell_{it}} = \left(1 - \tau_{it}^{m}\right) w_t, \tag{11}$$

$$\frac{\dot{\lambda}_{it}}{\lambda_{it}} = \beta_i + \delta - \left(1 - \tau_{it}^{m}\right) r_t, \tag{12}$$

$$\lim_{t \to \infty} \lambda_{it} K_{it} e^{-\beta_i t} = 0, \tag{13}$$

where λ_{it} is the co-state variable that characterizes the shadow (utility) value of physical capital. In addition, (11) equates the slope of an individual's indifference curve to the after-tax real wage rate, (12) is the standard consumption Euler equation and (13) is the transversality condition. After substituting (11) into (7), the capital accumulation equation for agent i can be written as

$$\dot{K}_{it} = (1-\tau_{it})(r_t K_{it} + w_t H_{it}) - \frac{(1-\tau_{it}^m) w_t (1-H_{it})}{\theta} - \delta K_{it}. \tag{14}$$

2.3 Government

The government spends its total tax revenues T_t on goods and services produced by competitive firms, and maintains a balanced budget at each instant of time. Hence, its instantaneous budget constraint is given by

$$G_t = T_t = \sum_{i=1}^{M} \tau_{it} Y_{it}, \tag{15}$$

where G_t represents public expenditures that are postulated to be a constant fraction of the economy's aggregate output:

$$G_t = gY_t, \quad 0 < g < 1, \tag{16}$$

where $Y_t \left(= M\overline{Y}_t\right)$ is given by (3). Finally, combining the aggregated version of (7) and equation (15) leads to the following economy-wide resource constraint:

$$C_t + I_t + G_t = Y_t, \tag{17}$$

where $C_t \left(= \sum_{i=1}^{M} C_{it}\right)$ denotes total consumption spending, and $I_t \left(= \sum_{i=1}^{M} \left[\dot{K}_{it} + \delta K_{it}\right]\right)$ represents total gross investment.

3 Long-Run Distribution of Income

This section first derives our model economy's equilibrium conditions, followed by examining the long-run balanced growth path (BGP) and the associated distributions of capital/wealth as well as income across all households. Before proceeding further, we note that the equalities of aggregate demand by a unit measure identical competitive firms versus aggregate supply by M groups of heterogenous agents in the capital and labor markets are given by

$$\int_0^1 K_{jt} dj = K_t = \sum_{i=1}^{M} K_{it}, \tag{18}$$

$$\int_0^1 H_{jt} dj = H_t = \sum_{i=1}^{M} H_{it}. \tag{19}$$

We also follow Koyuncu and Turnovsky (2016, p. 567) to define the notations:

$$\bar{\tau}_t \equiv \frac{1}{M}\sum_{i=1}^{M} \tau_{it}\left(\frac{Y_{it}}{\overline{Y}_t}\right) = \frac{\eta}{M}\sum_{i=1}^{M} y_{it}^{1+\phi}, \quad \text{where } \frac{1}{M}\sum_{i=1}^{M}\left(\frac{Y_{it}}{\overline{Y}_t}\right) = 1; \tag{20}$$

and

$$\bar{\tau}_t^m \equiv \frac{1}{M}\sum_{i=1}^{M} \tau_{it}^m\left(\frac{\ell_{it}}{\ell_t}\right) = \frac{\eta(1+\phi)}{M}\sum_{i=1}^{M} y_{it}^{\phi}\left(\frac{\ell_{it}}{\ell_t}\right), \quad \text{where } \frac{1}{M}\sum_{i=1}^{M}(\frac{\ell_{it}}{\ell_t}) = 1. \tag{21}$$

Equation (20) is a weighted average of the average tax rates levied on households, with the weights being their respective income levels relative to the economy-wide mean. Similarly, equation (21) is a weighted average of the corresponding individual marginal tax rates, with the weights being their respective relative leisure.

3.1 Macroeconomic Equilibrium

We first plug (3), (5) and (9) into the right-hand side of agent i's labor supply condition (11), and then take the time derivative to obtain

$$\frac{\dot{C}_{it}}{C_{it}} + \left(\frac{H_{it}}{1-H_{it}}\right)\frac{\dot{H}_{it}}{H_{it}} = \frac{\dot{K}_t}{K_t} - \left[\frac{\alpha(1-\varepsilon)}{\alpha+(1-\alpha)H_t^{\varepsilon}}\right]\frac{\dot{H}_t}{H_t} - \left[\frac{\eta(1+\phi)y_{it}^{\phi}}{1-\eta(1+\phi)y_{it}^{\phi}}\right]\frac{\dot{y}_{it}}{y_{it}}. \tag{22}$$

Given the equilibrium input prices (4)-(5), together with the definitions of the weighted tax rates (20)-(21) and the factor market clearing conditions (18)-(19), taking aggregation over each household's capital accumulation equation (14) yields that the economy-wide level of capital stock K_t will evolve over time according to

$$\frac{\dot{K}_t}{K_t} = A(1-\bar{\tau}_t)[\alpha+(1-\alpha)H_t^{\varepsilon}]^{\frac{1}{\varepsilon}} - \frac{A(1-\alpha)(1-\bar{\tau}_t^m)(1-H_{it})}{\theta}\left\{\frac{[\alpha+(1-\alpha)H_t^{\varepsilon}]^{\frac{1}{\varepsilon}}}{H_t}\right\}^{1-\varepsilon} - \delta. \tag{23}$$

After substituting equations (12) and (23) into condition (22), we can eliminate $\frac{\dot{C}_{it}}{C_{it}}$ and $\frac{\dot{K}_t}{K_t}$ from the above two equations to derive that for individual i,

$$\left(\frac{H_{it}}{1-H_{it}}\right)\frac{\dot{H}_{it}}{H_{it}} + \left[\frac{\alpha(1-\varepsilon)}{\alpha+(1-\alpha)H_t^{\varepsilon}}\right]\frac{\dot{H}_t}{H_t} + \left[\frac{\eta(1+\phi)y_{it}^{\phi}}{1-\eta(1+\phi)y_{it}^{\phi}}\right]\frac{\dot{y}_{it}}{y_{it}} - \frac{\beta_i+\gamma\delta}{1-\gamma}$$

$$= A[\alpha+(1-\alpha)H_t^{\varepsilon}]^{\frac{1}{\varepsilon}}\left\{1-\bar{\tau}_t - \frac{\alpha\left[1-\eta(1+\phi)y_{it}^{\phi}\right]}{(1-\gamma)[\alpha+(1-\alpha)H_t^{\varepsilon}]} - \frac{(1-\alpha)(1-\bar{\tau}_t^m)(1-H_t)}{\theta[\alpha+(1-\alpha)H_t^{\varepsilon}]H_t^{1-\varepsilon}}\right\}, \tag{24}$$

which will comprise a set of M independent equations.

Next, we use equations (14) and (23) to find that the evolution of household i's relative share of capital/wealth k_{it} $\left(\equiv \frac{K_{it}}{K_t}\right)$ is governed by

$$\dot{k}_{it} = A[\alpha+(1-\alpha)H_t^{\varepsilon}]^{\frac{1}{\varepsilon}} \left\{ (1-\tau_{it})y_{it} - \frac{(1-\alpha)(1-\tau_{it}^m)(1-H_{it})}{\theta[\alpha+(1-\alpha)H_t^{\varepsilon}]H_t^{1-\varepsilon}} - \left[1-\bar{\tau}_t - \frac{(1-\alpha)(1-\bar{\tau}_t^m)(1-H_t)}{\theta[\alpha+(1-\alpha)H_t^{\varepsilon}]H_t^{1-\varepsilon}}\right] k_{it} \right\}. \tag{25}$$

Finally, plugging the factor prices (4)-(5) into agent i's relative income share leads to

$$y_{it} \equiv \frac{Y_{it}}{\overline{Y}_t} = \frac{r_t K_{it} + w_t H_{it}}{r_t K_t + w_t H_t} = \left[\frac{\alpha}{\alpha+(1-\alpha)H_t^{\varepsilon}}\right] k_{it} + \left[\frac{(1-\alpha)H_t^{\varepsilon}}{\alpha+(1-\alpha)H_t^{\varepsilon}}\right] \frac{H_{it}}{H_t}. \tag{26}$$

Taking the time derivative of (26) then yields that

$$\dot{y}_{it} = \left[\frac{\alpha}{\alpha+(1-\alpha)H_t^{\varepsilon}}\right] \dot{k}_{it} + \left[\frac{(1-\alpha)H_t^{\varepsilon-1}}{\alpha+(1-\alpha)H_t^{\varepsilon}}\right] \dot{H}_{it} + \left\{ \frac{(1-\alpha)H_{it}H_t^{\varepsilon-2}[\alpha(\varepsilon-1)-(1-\alpha)H_t]}{[\alpha+(1-\alpha)H_t^{\varepsilon}]^2} \right\} \dot{H}_t. \tag{27}$$

Per the definitions of the weighted tax rates (20)-(21), equation (26) implies that

$$\frac{1}{M}\sum_{i=1}^{M} k_{it} = 1, \quad \frac{1}{M}\sum_{i=1}^{M} y_{it} = 1 \;\text{ and }\; \frac{1}{M}\sum_{i=1}^{M} H_{it} = H_t, \;\text{ for all }\; t. \tag{28}$$

As a result of this aggregate consistency condition (28), there are only $M-1$ accumulation equations for relative capital shares (25), as well as $M-1$ dynamical equations for relative income shares (27), that will be independent.

In sum, our model economy's equilibrium conditions consist of (i) M independent equations of (24), (ii) $M-1$ independent equations of (25), and (iii) $M-1$ independent equations of (27). It follows that they altogether will generate a dynamical system of $3M-2$ independent differential equations in terms of $\{H_{it}\}_{i=1}^{i=M}$ and $\{k_{it},\ y_{it}\}_{i=1}^{i=M-1}$.

3.2 Balanced Growth Path

We focus on the economy's long-run balanced growth path along which individual as well as aggregate labor hours, together with each household's relative shares of capital/wealth and income are stationary; whereas the economy-wide levels of output, consumption and physical capital all grow at a common constant rate $\tilde{\Phi} > 0$. By setting $\dot{H}_{it} = \dot{H}_t = \dot{k}_{it} = \dot{y}_{it} = 0$ in (24)-(25), coupled with the $M-1$ independent equations of (26) and the three aggregate-consistency conditions of (28), there are $3M+1$ interdependent equations that will be used to jointly determine $\tilde{H}$ and $\left\{\tilde{H}_i,\ \tilde{k}_i,\ \tilde{y}_i\right\}_{i=1}^{i=M}$ at the model's long-run equilibrium. Next, plugging $\tilde{y}_i$ into (8) and (20) leads to $\tilde{\tau}_i = \eta \tilde{y}_i^{\phi}$ and $\bar{\tau} = \frac{\eta}{M} \sum_{i=1}^{M} \tilde{y}_i^{1+\phi}$ along the balanced-growth equilibrium path, which in turn yields that household i's relative disposable-income share is given by $\tilde{y}_i^a = \frac{(1-\tilde{\tau}_i)}{(1-\bar{\tau})} \tilde{y}_i$.[5] We then follow Lerman and Yitzhaki (1989) to calculate the Gini coefficient on agents' steady-state before-tax income variability as follows:

$$Gini = 1 + \frac{1}{M} \left\{ 1 - \frac{2}{M} \left[\sum_{i=1}^{M} (M+1-i) \tilde{y}_i \right] \right\}. \tag{29}$$

Substituting $\tilde{y}_i^a$ into the above formula (29) generates the after-tax-income Gini coefficients denoted as $Gini^a$. Moreover, using the steady-state version of (25) and (26) results in the following relationship between $\tilde{y}_i$, $\tilde{H}_i$ and $\tilde{H}$:

$$\tilde{y}_i - (1-\alpha) \frac{\tilde{H}_i}{\tilde{H}} = \alpha \underbrace{\left[\frac{(1-\tilde{\tau}_i)\tilde{y}_i - \frac{(1-\alpha)(1-\tilde{\tau}_i^m)\left(1-\tilde{H}_i\right)}{\theta[\alpha+(1-\alpha)\tilde{H}^{\varepsilon}]\tilde{H}^{1-\varepsilon}}}{(1-\bar{\tau}) - \frac{(1-\alpha)(1-\bar{\tau}^m)(1-\tilde{H})}{\theta[\alpha+(1-\alpha)\tilde{H}^{\varepsilon}]\tilde{H}^{1-\varepsilon}}} \right]}_{\equiv \tilde{k}_i}, \tag{30}$$

where $\tilde{\tau}_i^m = \eta(1+\phi)\tilde{y}_i^{\phi}$ and $\bar{\tau}^m = \frac{\eta(1+\phi)}{M} \sum_{i=1}^{M} \tilde{y}_i^{\phi} \left(\frac{1-\tilde{H}_i}{1-\tilde{H}} \right)$.

On the other hand, we adopt (14) and (23) to obtain that the time-varying growth rates of individual and aggregate capital stocks are given by

$$\Phi_{K_{it}} \equiv \frac{\dot{K}_{it}}{K_{it}} = \frac{A[\alpha + (1-\alpha)H_t^{\varepsilon}]^{\frac{1}{\varepsilon}}}{k_{it}} \left[(1-\tau_{it})y_{it} - \frac{(1-\alpha)\left(1-\tau_{it}^m\right)(1-H_{it})}{\theta[\alpha+(1-\alpha)H_t^{\varepsilon}]H_t^{1-\varepsilon}} \right] - \delta, \tag{31}$$

and

$$\Phi_{K_t} \equiv \frac{\dot{K}_t}{K_t} = A[\alpha + (1-\alpha)H_t^{\varepsilon}]^{\frac{1}{\varepsilon}} \left[(1-\bar{\tau}_t) - \frac{(1-\alpha)\left(1-\bar{\tau}_t^m\right)(1-H_t)}{\theta[\alpha+(1-\alpha)H_t^{\varepsilon}]H_t^{1-\varepsilon}} \right] - \delta. \tag{32}$$

Since H_t, k_{it} and y_{it} (along with the resulting τ_{it}, τ_{it}^m, $\bar{\tau}_t$ and $\bar{\tau}_t^m$) are all fixed constants at our model's stationary equilibrium, it is straightforward to show that $\Phi_{K_{it}}$ and Φ_{K_t} will converge to a common growth rate as time moves forward. In addition, we note that $\dot{y}_{it} = 0$ leads to $\Phi_{Y_{it}} = \Phi_{\overline{Y}_t} = \Phi_{Y_t}$; and that taking the time derivative of the social technology (3) generates $\Phi_{Y_t} = \Phi_{K_t}$. From equation (22) with $\dot{H}_{it} = \dot{H}_t = \dot{y}_{it} = 0$, we also find that $\Phi_{C_{it}} = \Phi_{K_t}$ along

[5] Koyuncu and Turnovsky (2016, p. 586) define agent i's relative after-tax income share as $\tilde{y}_i^a = (1-\tilde{\tau}_i)\tilde{y}_i$. This in turn implies that $\frac{1}{M} \sum_{i=1}^{M} \tilde{y}_i^a$ is not equal to one, which is inconsistent with the OECD panel dataset used for our model calibrations.

the economy's balanced growth path. Per condition (11) and the equality $C_t = \sum_{i=1}^{M} C_{it}$, it can be shown that individual i's relative share of consumption is

$$c_{it} \equiv \frac{C_{it}}{C_t} = \left(\frac{1-\tau_{it}^m}{1-\bar{\tau}_t^m}\right)\left(\frac{1-H_t}{1-H_{it}}\right); \tag{33}$$

and that the associated stationary or long-run expression is given by

$$\tilde{c}_i = \frac{1-\eta(1+\phi)\tilde{y}_i^{\phi}}{1-\frac{\eta(1+\phi)}{M}\sum_{i=1}^{M}\tilde{y}_i^{\phi}\left(\frac{1-\tilde{H}_i}{1-\tilde{H}}\right)}\left(\frac{1-\tilde{H}}{1-\tilde{H}_i}\right). \tag{34}$$

Given the BGP levels of $\tilde{y}_i$, $\tilde{H}_i$ and $\tilde{H}$ are time invariant, the resulting constancy of $\tilde{c}_i$ implies that $\Phi_{C_{it}} = \Phi_{C_t}$. Finally, we use the steady-state consumption Euler equation (12) to derive the common (positive) rate of economic growth $\tilde{\Phi}\left(= \tilde{\Phi}_{\tilde{Y}} = \tilde{\Phi}_{\tilde{C}} = \tilde{\Phi}_{\tilde{K}} = \tilde{\Phi}_{\tilde{Y}_i} = \tilde{\Phi}_{\tilde{C}_i} = \tilde{\Phi}_{\tilde{K}_i}\right)$ at the model's unique balanced-growth equilibrium:

$$\tilde{\Phi} = \frac{1}{1-\gamma}\left\{\underbrace{\alpha A[1-\eta(1+\phi)\tilde{y}_i^{\phi}][\alpha+(1-\alpha)\tilde{H}^{\varepsilon}]^{\frac{1-\varepsilon}{\varepsilon}}}_{=\,(1-\tilde{\tau}_i^m)\tilde{r}} - \beta_i - \delta\right\}, \tag{35}$$

where $\tilde{r}$ is the stationary level of capital rental rate.

3.3 Comparative Statics in Tax Progressivity

Under a given degree of income-tax progressivity $\phi \in (0, \min\{1, \frac{1-\eta}{\eta}\})$, our comparative statics analysis begins with examining how differential rates of time preference affect the model's long-run individual levels of labor hours, as well as their relative gross/net income shares and capital/wealth ownership.

Proposition 1. In our endogenously growing macroeconomy with a monotonically decreasing sequence of time preference rates $\beta_1 > \beta_2 > \cdots > \beta_M$, relatively more patient agents will (i) work less $\tilde{H}_M < \tilde{H}_{M-1} < \cdots < \tilde{H}_1$, (ii) have higher shares of before-tax as well as after-tax income: $\tilde{y}_M > \tilde{y}_{M-1} > \cdots > \tilde{y}_1$ along with $\tilde{y}_M^a > \tilde{y}_{M-1}^a > \cdots > \tilde{y}_1^a$, and (iii) own higher shares of capital/wealth $\tilde{k}_M > \tilde{k}_{M-1} > \cdots > \tilde{k}_1$.

Proof. This Proposition can be proved by first considering any pair of households i and j with $\beta_i > \beta_j$, Appendix A analytically shows that $\tilde{H}_j < \tilde{H}_i$, $\tilde{y}_j > \tilde{y}_i$, $\tilde{y}_j^a > \tilde{y}_i^a$ and $\tilde{k}_j > \tilde{k}_i$. It is then straightforward to generalize this bilateral result to an economy with $M \geq 3$ groups of agents and $\beta_1 > \beta_2 > \cdots > \beta_M$.■

Proposition 1 illustrates that heterogeneity in agents' time preference rates plays an important role in determining the macroeconomy's equilibrium allocations. Since the BGP growth rate of output (35) is identical across all individuals, it is immediately clear that any pair of households with $\beta_i = \beta_j$ will face the same stationary marginal tax rate $\left(\tilde{\tau}_i^m = \tilde{\tau}_j^m\right)$, which in turn leads to a perfectly equal distribution of income, capital/wealth and hours worked, even though their initial capital endowments are different. As in Li and Sarte (2004) or Koyuncu and Turnovsky (2016), Appendix A shows that differences in discount rates $\left(\beta_i \neq \beta_j\right)$ will generate non-degenerate dispersions in key macroeconomic variables because progressive income taxation entails differentiated after-tax returns to agents' capital investment. In particular, since the more patient household j has a relatively lower marginal utility of wealth, it will choose to supply less labor $\left(\tilde{H}_j < \tilde{H}_i\right)$. This result, coupled with $\tilde{y}_j > \tilde{y}_i$ from (A.1), implies that agent j's steady-state capital/wealth share is higher $\left(\tilde{k}_j > \tilde{k}_i\right)$ as shown in equation (A.7). Under flat income taxation with $\phi = 0$, we recover Becker's (1980) canonical finding that the most patient agent will hold the economy's entire capital stock along its balanced growth path.

Next, we examine the distributional and growth effects of a change in the fiscal progression within a simplified two-type (patient versus impatient) version of the model economy.

Proposition 2. In our endogenously growing macroeconomy with $M = 2$ groups of individuals and $\beta_1 > \beta_2$, an increase in the tax progressivity ϕ will lead to (i) decreases in the pre-tax as well as after-tax income inequality, and (ii) a lower balanced growth rate of output, consumption and capital, provided the elasticity of aggregate labor hours with respect to the progressive level of income taxation E_ϕ^H exceeds a specific (negative) threshold.

Proof. Equation (B.2) shows that when the sufficient condition $E_\phi^H > \underline{E} \equiv -\frac{[\alpha+(1-\alpha)\tilde{H}^\varepsilon]\phi}{(1-\alpha)(1-\varepsilon)(1+\phi)\tilde{H}^\varepsilon}$ is satisfied, impatient households' relative gross income share will rise in response to more progressive taxation $\left(\frac{\partial \tilde{y}_1}{\partial \phi} > 0\right)$. As this requisite condition holds, we then prove that $\frac{\partial Gini}{\partial \phi} < 0$ per equation (B.3); $\frac{\partial Gini^a}{\partial \phi} < 0$ per equation (B.5); and $\frac{\partial \tilde{\Phi}}{\partial \phi} < 0$ per equation (B.6).■

Under our postulated homogenous preference formulation (6), an increase in the tax progressivity leads to a lower economy-wide level of hours worked due to a stronger substitution (*c.f.* income) effect, thus $E_\phi^H < 0$ which in turn *ceteris paribus* decreases the right-hand-side of equation (B.1). In order to maintain the same difference between agents' time preference rates per the left-hand-side of (B.1), $\tilde{y}_2^\phi - \tilde{y}_1^\phi$ needs to rise for balancing out the reduction in $\tilde{H}$. Since $\tilde{y}_1 < 1 < \tilde{y}_2$ plus $\frac{\partial \tilde{y}_1}{\partial \phi}$ and $\frac{\partial \tilde{y}_2}{\partial \phi}$ are of opposite signs, this can be achieved if the condition $E_\phi^H > \underline{E}$ holds to raise type-1 individuals' relative before-tax income share, as shown by equation (B.2). It follows that the economy's long-run distribution of gross income shares will become less dispersed with a higher degree of fiscal progression, hence $\frac{\partial Gini}{\partial \phi} < 0$ *à la* (B.3). Using $\tilde{\tau}_i = \eta \tilde{y}_i^{\phi}$ yields that while keeping $\tilde{y}_1$ and $\tilde{y}_2$ unchanged, the direct effect of a higher level of ϕ is that the average tax rate faced by impatient households falls, whereas that for patient individuals rises. This result, together with $\frac{\partial \tilde{y}_1}{\partial \phi} > 0$ and $\frac{\partial \tilde{y}_2}{\partial \phi} < 0$, helps explain the decreases in the differential between agents' relative after-tax income shares as well as the ensuing net-income inequality per equations (B.4) and (B.5), respectively. Finally, equation (B.6) finds that the economy's equilibrium output growth rate will decrease/increase when the tax schedule becomes more/less progressive $\left(\frac{\partial \tilde{\Phi}}{\partial \phi} < 0\right)$ because of a lower/higher post-tax rate of return on capital investment.

Since the BGP quantities of $\tilde{H}$ and $\left\{\tilde{H}_i,\ \tilde{k}_i,\ \tilde{y}_i\right\}_{i=1}^{i=M}$ are jointly determined by $3M+1$ interdependent equations, their dynamic interrelations with $M \geq 3$ are rather complicated and not analytically tractable. For such an environment, the next section will undertake a quantitative assessment on the long-run macroeconomic effects of progressive income taxation within a calibrated version of our one-sector endogenous growth model.

4 Quantitative Analysis

As in Li and Sarte (2004), we consider five heterogeneous types of households $M = 5$ with a monotonically decreasing sequence of time preference rates β_i, where $i = 1, 2, \ldots 5$. Using the solution procedure described in subsection 3.2, the economy's stationary equilibrium levels of $\tilde{H}$ and $\left\{\tilde{H}_i,\ \tilde{k}_i,\ \tilde{y}_i\right\}_{i=1}^{i=5}$ are determined by simultaneously solving the steady-state versions of (i) five independent equations of (24), (ii) four independent equations of (25), (iii) four independent equations of (27) and three aggregate-consistency conditions of (28). The remaining endogenous variables along our model's unique balanced growth path, such as $\{\tilde{\tau}_i,\ \tilde{\tau}_i^m,\ \tilde{y}_i^a,\ \tilde{c}_i\}_{i=1}^{i=5}$ and $\left\{\bar{\tau},\ \bar{\tau}^m,\ \tilde{\Phi}\right\}$, can then be obtained accordingly. Since Proposition 1 states that relatively more patient households have higher steady-state equilibrium shares of pre-tax as well as post-tax income, the long-run quintile distributions of gross income $\left\{\frac{\tilde{y}_i}{M}\right\}_{i=1}^{i=5}$ and net income $\left\{\frac{\tilde{y}_i^a}{M}\right\}_{i=1}^{i=5}$, generated from our postulated declining sequence of β_i, will be characterized by the first quintile $\left(\frac{\tilde{y}_1}{M} \text{ or } \frac{\tilde{y}_1^a}{M}\right)$ being the lowest and the fifth quintile $\left(\frac{\tilde{y}_5}{M} \text{ or } \frac{\tilde{y}_5^a}{M}\right)$ being the highest. With these quintile distributions, we will use (29) to calculate the model's long-run income inequality measures given by $Gini$ and $Gini^a$.

4.1 Benchmark Calibration

Our model macroeconomy is postulated to begin at a balanced-growth equilibrium with endogenously-determined $\tilde{H}$ and $\left\{\tilde{H}_i,\ \tilde{k}_i,\ \tilde{y}_i\right\}_{i=1}^{i=5}$. For the benchmark parameterization, the capital depreciation rate δ is 0.025; the technological parameter $\alpha = 0.4948$ is set to generate the steady-state capital share of national income at 0.45; and the preference parameter $\theta = 1.4325$ is chosen to yield that the initial level of economy-wide hours worked is 0.3. With regard to the intertemporal elasticity of substitution in (effective) consumption $IES = \frac{1}{1-\gamma}$, many existing studies have adopted the range of one-third to one in their quantitative analyses. However, some empirical research suggests that $IES > 1$ and thus $\gamma \in (0,\ 1)$. For example, Vissing-Jørgensen and Attanasio (2003) report the point estimates of IES to be 1.03 (with six instruments) and 1.44 (under one instrumental variable) for the group of all stock holders; and Gruber (2006) presents that the estimated elasticity of intertemporal consumption substitution is around 2 when endogenous tax rate variations are included in his cross-sectional estimation on U.S. total non-durable consumption expenditures. Based on these results, we calibrate $\gamma = -2$ to be the baseline value, which leads to the lowest possible IES $\left(= \frac{1}{3}\right)$ that is regarded as empirically plausible. On the other hand, Knoblach et al. (2020) carry out a meta-regression analysis to synthesize empirical findings from 77 econometric studies with more than 2,400 estimates on the elasticity of substitution between productive capital and labor $ELS = \frac{1}{1-\varepsilon}$, and report an estimated range of $0.45 - 0.87$ for the aggregate economy. Another recent article by Gechert et al. (2022) finds that the mean implied capital-labor substitution elasticity is 0.3, after controlling for publication bias and model uncertainty presented in 121 previous empirical studies. Drawing upon these results, we choose $ELS = 0.87$ or ($\varepsilon = -0.1494$) as the benchmark calibration, which represents a non-negligible departure from the conventional Cobb-Douglas specification *à la* Koyuncu and Turnovsky (2016).

Our calibrations of the tax-schedule parameters η and ϕ are based on the OECD Income Distribution Database (2020) over the $2008 - 2017$ period. For each period t, we take simple averages across 32 countries on their income tax rates along with quintile distributions of gross income shares as well as disposable income (post taxes and transfers) shares to construct τ_t and $\{y_{it},\ y_{it}^a\}_{i=1}^{i=5}$. We then substitute $\tau_{it} = \eta y_{it}^{\phi}$ into $y_{it}^a = \frac{(1-\tau_{it})}{(1-\tau_t)} y_{it}$ and take logarithm to obtain

$$\log\left[1 - (1-\tau_t)\,\frac{y_{it}^a}{y_{it}}\right] = \log\eta + \phi\log y_{it}. \tag{36}$$

Ordinary least squares regression results in the point estimates of $\eta = 0.247$ and $\phi = 0.3744$. Using the time averages of pre-tax income shares $\tilde{y}_i = \frac{1}{T}\sum_{t=1}^{T} y_{it}$ to denote the model's initial steady-state condition, together with estimated values of $\{\eta,\ \phi\}$ and the stationary version of (20), yields that the calibrated average tax rate for our aggregate economy is equal to

$$\bar{\tau} = \frac{1}{5}\sum_{i=1}^{5}\tau_i\tilde{y}_i = \frac{\eta}{5}\sum_{i=1}^{5}\tilde{y}_i^{1+\phi} = 0.2686. \tag{37}$$

Based on a dataset of national tax schedules collected for 189 countries from 1981 to 2005, Duncan and Sabirianova Peter (2016, Table 1) report a mean value of average rate progression $ARP \equiv \frac{\bar{\tau}^m - \bar{\tau}}{\bar{\tau}} = 0.0362$, which in turn implies that the model's average marginal tax rate is set to $\bar{\tau}^m = 0.2783$.

We calibrate agents' time preference rates β_i to match the model's beginning quintile distribution of after-tax income shares (using the time averages $\tilde{y}_i^a = \frac{1}{T}\sum_{t=1}^{T} y_{it}^a$) with the actual data from our sample OECD countries whereby $\left\{\frac{\tilde{y}_1^a}{5}, \frac{\tilde{y}_2^a}{5}, \frac{\tilde{y}_3^a}{5}, \frac{\tilde{y}_4^a}{5}, \frac{\tilde{y}_5^a}{5}\right\} = \{8.05\%, 13.29\%, 17.45\%, 22.75\%, 38.46\}$. This procedure gives rise to $\beta_1 = 0.0987$, $\beta_2 = 0.0862$, $\beta_3 = 0.078$, $\beta_4 = 0.0689$ and $\beta_5 = 0.0469$. As a result, the gross- and disposable-income inequality measures are $Gini = 0.3177$ and $Gini^a = 0.2812$, respectively, at our economy's original stationary equilibrium. Finally, the technological parameter A is set to be 1.0005 such that the starting BGP output growth rate $\tilde{\Phi} = 2\%$.

4.2 Baseline Results

Given the above-mentioned benchmark values of model parameters, Table 1 presents the long-run effects on agents' relative shares of capital/wealth and after-tax income, the individual/aggregate labor hours, as well as the output growth rate and the net-income inequality of a one-percentage-point increase in the average rate progression. The "$ARP = 0.0362$ ($\phi = 0.3744$)" columns present the beginning levels of these variables at the economy's original balanced-growth equilibrium; whereas the "$ARP' = 0.0462$ ($\phi' = 0.381$)" columns report the corresponding values under a higher degree of tax progressivity. Notice that the relative capital shares for the lowest two quintiles of individuals are below zero.[6] Using the steady-state version of equation (26), it is straightforward to show that

$$\tilde{k}_i = \underbrace{\left(\frac{\alpha + (1-\alpha)\tilde{H}^{\varepsilon}}{\alpha}\right)}_{\text{Positive}}\left[\tilde{y}_i - \frac{(1-\alpha)\tilde{H}^{\varepsilon}}{\alpha + (1-\alpha)\tilde{H}^{\varepsilon}}\frac{\tilde{H}_i}{\tilde{H}}\right], \tag{38}$$

where $\frac{(1-\alpha)\tilde{H}^{\varepsilon}}{\alpha+(1-\alpha)\tilde{H}^{\varepsilon}} = 0.55$ is the labor share of national income under our baseline parameterization. Since $\tilde{y}_1 = 0.3535$ and $\tilde{y}_2 = 0.6117$, together with $\tilde{H}_1 = 0.5692$ and $\tilde{H}_2 = 0.4333$,

[6] According to the OECD Wealth Distribution Database (2022), net worth for the bottom 20% households in the following countries/years were negative: Chile in 2017, Denmark in 2019, Finland in 2016, Germany in 2017, Greece in 2018, Japan in 2019, Netherlands in 2019, Norway in 2018, and the United States in 2019.

the bracket term on the right-hand of (38) will be negative for $i = 1$ and 2.[7] Although these poorest households do not own any capital stock, their total factor income $\tilde{Y}_1$ and $\tilde{Y}_2$ turns out to be positive due to relatively high supply of labor hours. On the other hand, the top quintile of individuals begin with owning 92.46% of the economy's capital stock and providing a rather small amount of labor supply because of their low marginal utility of income. We also verified that given the benchmark parametric configuration, the requisite sufficient condition for $\frac{\partial \tilde{y}_1}{\partial \phi} > 0$ as in (B.2) is satisfied since the general-equilibrium elasticity E_ϕ^H $(= -0.01026)$ is larger than $\underline{E}(= -0.43088)$.

When the tax schedule becomes more progressive, (as discussed earlier) the economy-wide labor hours $\tilde{H}$ will fall and thus raising the wage-rate function $\omega\left(\tilde{H}\right)$ given by (5). While keeping other things being equal, higher fiscal progression leads to immediate increases/decreases in the post-tax real wages faced by the first-three/remaining-two groups of agents $(1 - \tilde{\tau}_i)\,\omega\left(\tilde{H}\right)$ because $\tilde{y}_1 < \tilde{y}_2 < \tilde{y}_3 < 1 < \tilde{y}_4 < \tilde{y}_5$. It follows that on impact, the first/second/middle quintile of households will increase their hours worked, whereas the labor supply of the fourth/highest quintile of individuals will decline. During the transition instants of time, the first three quintiles of capital-poor households choose to work less; and the highest two quintiles of capital-rich individuals supply more hours worked. At the economy's new stationary equilibrium, we find that labor hours are lower for the first four quintiles of agents, but higher for the most patient group, than those of their initial levels.

Using equation (4), a lower level of $\tilde{H}$ also yields a reduction in the rate of return to capital investment $\tilde{r}$. This decrease in turn generates more adverse effects on the top two quintiles of agents since their income tax rates are negatively affected by a higher degree of tax progressivity. As a result, the rate of these wealthiest individuals' capital accumulation will be reduced immediately. By contrast, the remaining three quintiles of households will raise their investment expenditures on impact due to higher post-tax capital rental rates $\{(1 - \tilde{\tau}_i)\,\tilde{r}\}_{i\,=\,1}^{i\,=\,3}$. Along the transition path, the first three quintiles of capital-poor agents slow down their capital accumulation rate; whereas the highest two quintiles of capital-rich households accumulate their wealth at a faster rate. In the long run, we find that $\frac{\tilde{k}_5}{5}$ falls to 91.22% and that $\left\{\frac{\tilde{k}_i}{5}\right\}_{i\,=\,1}^{i\,=\,4}$ become higher at the model's new balanced-growth equilibrium.

In light of the preceding discussion, the new steady-state distributions of agents' relative capital/wealth shares and labor hours are less dispersed than their original counterparts. It follows that the long-run distributional consequence of more progressive taxation on their pre-

[7] In a one-sector endogenous growth model with infinitely-lived households differentiated by capital endowments and intertemporal elasticities of substitution, Carneiro et al. (2022) examine the interrelations between the growth-inequality tradeoff and the degree of tax progressivity. In their calibrated initial balanced-growth equilibrium, the relative capital share for the bottom quintile of agents is −3.92%.

tax or post-tax income will be a lower degree of inequality, as per Proposition 2. However, the resulting reduction in disposable-income inequality turns out to be quantitatively insignificant. Under our benchmark parameterization, Table 1 shows that a one-percentage-point increase in the average rate progression yields a decrease in $Gini^a$ by 1.92%, which in turn leads to a calculated elasticity (in absolute value) of 0.0693.[8] This figure is substantially lower than the estimated range of $0.1206 - 0.411$ reported by Duncan and Sabirianova Peter (2016). Finally, we note that a higher tax progressivity will decrease the economy's output growth rate $\tilde{\Phi}$ to 1.98% because of a smaller after-marginal-tax rate of return on capital investment, *i.e.* $\frac{\partial\left[(1-\tilde{\tau}_i^m)\tilde{r}\right]}{\partial\phi} < 0$ for all i (see equation 35).

4.3 Sensitivity Analysis

With regard to the sensitivity analysis, Table 2 presents after-tax income inequality measures and the ensuing calculated elasticities with respect to a change in fiscal progression of our macroeconomy under alternative empirically-plausible calibrations: $IES = \frac{1}{3}$, or $IES = 1$ whereby the instantaneous utility function (6) is separable and logarithmic in consumption and leisure, or $IES = 2$; along with $ELS = 0.87$; or $ELS = 0.45$; or $ELS = 0.3$. In order to maintain comparability, other model parameters are re-calibrated to achieve that each parametric configuration starts at the same balanced-growth equilibrium with $Gini^a = 0.2812$ under $ARP = 0.0362$.

We find that an increase in agents' intertemporal elasticity of consumption substitution strengthens the steady-state (negative) dispersion responses of their gross/disposable income under more progressive taxation. Intuitively, a higher γ enables individuals to more readily adjust consumption expenditures across different instants of time and generates a larger reduction in the interest rate on impact. This effect, coupled with a higher degree of tax progressivity, will speed up (slow down) capital-poor (capital-rich) households' wealth accumulation rate along the transition path. As a result, Table 2 shows that in the "$ARP = 0.0462$" columns, the after-tax-income Gini coefficient is *ceteris paribus* monotonically decreasing with respect to the intertemporal elasticity of substitution in consumption $\left(\frac{\partial Gini^a}{\partial IES} < 0\right)$. However, for our benchmark specification with $ELS = 0.87$, the resulting calculated elasticities (0.07801 when $IES = 1$; and 0.1051 when $IES = 2$) are still unrealistically low *vis-à-vis* panel estimation results of previous econometric studies.[9]

[8] The corresponding gross-income *Gini* coefficient will decline from 0.3177 when ARP = 0.0362 to 0.3124 when ARP = 0.0462, resulting in a calculated elasticity of 0.0603.

[9] When firms' production technology takes on the conventional Cobb-Douglas formulation with $ELS = 1$ *à la* Koyuncu and Turnovsky (2016), the calculated elasticity is equal to 0.06943 under $IES = \frac{1}{3}$; 0.07766 under $IES = 1$; and 0.10153 under $IES = 2$. None of these numerical results is a close match with the empirical

We also find that when the fiscal policy rule becomes more progressive, a decrease in ELS between capital and labor inputs can either raise or reduce the economy's disposable-income inequality. For a lower level of elasticity of factor substitution, the less flexible production technology yields a larger increase in the wage-rate function $\omega\left(\tilde{H}\right)$ on impact which in turn produces two opposite effects. On the one hand, faster wage growth during the subsequent transition causes capital-poor households to dis-save more for boosting their current consumption, resulting in a more unequal long-run wealth distribution. On the other hand, the preceding effect is countered by the labor supply responses of these households since higher future wages tend to increase both current consumption and future leisure. The desire to work less in the future will moderate capital-poor individuals' incentive to reduce their capital accumulation rate, leading to a less dispersed wealth distribution. In addition, decreasing ELS generates a larger reduction in the return to capital investment $\tilde{r}$. This outcome will then lower the capital share of national income because of a smaller steady-state capital-to-labor ratio, and thus making the wealth distribution less unequal. Taken together, it follows that whether a change in the elasticity of factor substitution amplifies or mitigates the decline in after-tax income inequality under more progressive taxation depends on the the relative strength of quantitative impacts on three key factors mentioned above: capital/wealth dispersion, labor supply, and factor income shares. Table 2 shows that when agents' intertemporal elasticity of consumption substitution is equal to 1 (the log-log utility function) or 2, the Gini coefficient on after-tax income is *ceteris paribus* monotonically increasing with respect to the elasticity of substitution in production $\left(\frac{\partial Gini^a}{\partial ELS} > 0\right)$; and that the reverse $\frac{\partial Gini^a}{\partial ELS} < 0$ holds true when $IES = \frac{1}{3}$. Finally, when the intertemporal elasticity of substitution in consumption takes on the highest possible value that is regarded as empirically realistic $IES = 2$, together with $ELS = 0.45$, a one-percentage-point increase in the average rate progression leads to a decrease in $Gini^a$ by 3.9118%. This will result in a calculated elasticity of 0.1404, which lies above the lower bound of the estimated interval [0.1206, 0.411] that Duncan and Sabirianova Peter (2016) have obtained.

5 Useful Government Spending

In the context of our baseline model with progressive income taxation and sustained long-run growth analyzed above, government purchases are postulated to be wasteful because they do not contribute to firms' production or agents' utility. This assumption, although commonly adopted by numerous previous theoretical studies for the sake of analytical simplicity, is not

evidence.

necessarily the most realistic *vis-à-vis* those observed in U.S. and many industrialized countries. In this section, we will examine an identical endogenously growing macroeconomy, but with useful public expenditures on goods and services. On the economy's supply side, government spending may enter the representative firm's production technology (1) as a positive externality that is complementary to aggregate capital stock *à la* Chatterjee and Turnovsky (2012). On the economy's demand side, public spending may enter household i's preference formulation (6) non-separably as a positive preference externality *à la* García-Peñalosa and Turnovsky (2011). In what follows, numerical experiments are undertaken to quantitatively gauge the long-run distributional effects on individuals' disposable income under either productive or utility-generating government purchases within our model economy.

5.1 Productive Government Spending

In this case, the representative firm j produces output Y_{jt} according to the modified CES production function given by

$$Y_{jt} = A\left[\alpha K_{jt}^{\varepsilon} + (1-\alpha)(K_t^{\chi} G_t^{1-\chi} H_{jt})^{\varepsilon}\right]^{\frac{1}{\varepsilon}}, \ A > 0, \ \ 0 < \alpha, \ \chi < 1 \ \text{ and } \ -\infty < \varepsilon \leqslant 1, \quad (39)$$

where $K_t^{\chi} G_t^{1-\chi}$ represents a composite externality that depends on the aggregate capital stock and the flow of productivity-enhancing government expenditures. It follows that the economy's social technology now becomes

$$Y_t = A[\alpha + (1-\alpha)\left(z_t^{1-\chi} H_t\right)^{\varepsilon}]^{\frac{1}{\varepsilon}} K_t, \quad (40)$$

where $z_t \equiv \frac{G_t}{K_t}$; and that the first-order conditions for each firm's profit maximization problem are changed to

$$r_t = \alpha A[\alpha + (1-\alpha)\left(z_t^{1-\chi} H_t\right)^{\varepsilon}]^{\frac{1-\varepsilon}{\varepsilon}}, \quad (41)$$

$$w_t = \underbrace{(1-\alpha)A[\alpha + (1-\alpha)\left(z_t^{1-\chi} H_t\right)^{\varepsilon}]^{\frac{1-\varepsilon}{\varepsilon}} z_t^{\varepsilon(1-\chi)} H_t^{\varepsilon-1}}_{\equiv\, \omega(z_t,\, H_t)} K_t. \quad (42)$$

Next, we follow the same solution procedure as in section 3 to derive the model's balanced growth path with endogenously-determined $\left\{\tilde{H},\ \tilde{z}\right\}$ and $\left\{\tilde{H}_i,\ \tilde{k}_i,\ \tilde{y}_i\right\}_{i\,=\,1}^{i\,=\,5}$.

In terms of the numerical experiments, we follow Chatterjee and Turnovsky (2012) to set the level of productive externalities $\chi = 0.6$ and calibrate the remaining model parameters such that $Gini^a = 0.2812$ with $ARP = 0.0362$ results at the initial stationary equilibrium. Table 3 presents the net-income inequality measures and calculated elasticities under $ARP = 0.0462$ when the households' $IES = \{\frac{1}{3}, 1, 2\}$ together with the firms' $ELS = \{0.87, 0.45, 0.3\}$, for ease of comparative comparisons with Table 2.

We first find that when agents' IES is equal to one or two, the after-tax Gini coefficients are lower and the resulting calculated elasticities are higher (in absolute terms) in Table 3 with $\chi = 0.6$ than those corresponding to Table 2 with $\chi = 0$, for all values of elasticity of capital-labor substitution under consideration. Intuitively, incorporating productive public spending leads to a larger increase in the wage-rate function $\omega(\cdot)$ and reduction in the interest rate on impact of more progressive taxation. When the household preference formulation is logarithmically separable or exhibits less risk aversion, these immediate effects will prompt capital-poor individuals to decrease their future labor supply at a faster rate as well as yield a greater decline in the steady-state capital share of national income, both of which result in a less unequal long-run distribution of disposable income. In particular, the calculated elasticity of post-tax Gini with respect to the average rate progression is 0.12777 under the parametric configuration with $IES = 2$ and $ELS = 0.87$. This figure turns out to be slightly above the lower bound of Duncan and Sabirianova Peter's (2016) estimated interval [0.1206, 0.411]. We also note that when the intertemporal elasticity of consumption substitution is low $\left(IES = \frac{1}{3}\right)$, the inclusion of productive government purchases does not exert quantitatively significant impact on the economy's aggregate distributional dynamics. It follows that the numerical results on $Gini^a$ and calculated elasticities reported in Tables 2 and 3 are essentially identical for this parameterization.

5.2 Utility-Generating Government Spending

In this case, agent i's discounted lifetime utilities are modified to

$$\int_0^\infty \underbrace{\frac{1}{\gamma}\left(C_{it}\ell_{it}^{\theta}G_t^{\mu}\right)^{\gamma}}_{\equiv U_{it}} e^{-\beta_i t}dt, \; -\infty < \gamma < 1, \quad \theta, \; \mu > 0, \; \text{and} \; \; \gamma\theta < 1, \tag{43}$$

where μ represents the degree of a positive preference externality that public spending yields on "effective consumption" $C_{it}\ell_{it}^{\theta}$. It follows that the co-state variable which characterizes the shadow (utility) value of physical capital becomes

$$C_{it}^{\gamma-1}\ell_{it}^{\gamma\theta}G_t^{\gamma\mu} = \lambda_{it}. \tag{44}$$

When $\gamma = 0$, the instantaneous utility function U_{it} exhibits additive separability between private consumption, leisure and public good, hence the marginal utilities of C_t and ℓ_t are independent of G_t. When $\gamma > (<)\ 0$, the marginal utility of private consumption or leisure increases (decreases) with respect to government purchases, thus C_t (or ℓ_t) and G_t are Edgeworth complements (substitutes). It is then straightforward to derive the economy's unique balanced-growth path along which aggregate output will grow at the rate of

$$\tilde{\Phi} = \frac{1}{1-\gamma(1+\mu)} \left\{ \underbrace{\alpha A[1-\eta(1+\phi)\tilde{y}_i^{\phi}][\alpha + (1-\alpha)\tilde{H}^{\varepsilon}]^{\frac{1-\varepsilon}{\varepsilon}}}_{=(1-\tilde{\tau}_i^m)\tilde{r}} - \beta_i - \delta \right\}. \tag{45}$$

Using García-Peñalosa and Turnovsky's (2011) calibration of $\mu = 0.3$, Table 4 presents the associated dispersion/inequality effects for our endogenously growing macroeconomy that starts at the common $Gini^a = 0.2812$ under $ARP = 0.0362$, in conjunction $IES = \{\frac{1}{3},\, 1,\, 2\}$ and $ELS = \{0.87,\, 0.45,\, 0.3\}$. We first note that when the household preference formulation (43) is logarithmically separable in private and public consumption goods ($\gamma = 0$), the inclusion of utility-enhancing public expenditures does not generate any impact on the model's equilibrium conditions and distributional dynamics. As a result, the quantitative results reported in Tables 2 and 4 are identical when agents' intertemporal elasticity of consumption substitution is equal to 1.

Table 4 also shows that when $IES = \frac{1}{3}$ or $\gamma = -2$, the after-tax Gini coefficients are higher under $\mu = 0.3$ than those corresponding to Table 2 with $\mu = 0$. In this environment with C_t (or ℓ_t) and G_t as Edgeworth substitutes, a decrease in public spending under more progressive taxation leads to increases in the marginal utilities of private consumption and leisure, which in turn will induce capital-poor individuals to slow down the reductions in their labor supply and capital accumulation during the subsequent transition. Consequently, the long-run distribution of capital/wealth is more dispersed and the resulting calculated elasticities of $Gini^a$ with respect to the average rate progression are smaller than those in Table 2 for our benchmark model. The opposite intuitive explanation is applicable to the setting with $IES = 2$ or $\gamma = 0.5$, as private consumption (or leisure) and public good are now preference complements. In response to a higher tax progressivity that leads to lower government purchases, capital-poor agents speed up the decreases in their hours worked and investment expenditures along the transition path, hence the economy's stationary equilibrium distribution of post-tax income will be less unequal. Under the baseline parameterization of $ELS = 0.87$, the calculated elasticity is raised to 0.1915 which turns out to be significantly larger than the lower bound of the empirically-plausible interval [0.1206, 0.411].

Overall, the preceding numerical simulations illustrate that when agents' intertemporal elasticity of consumption substitution takes on the highest possible value that is regarded as empirically plausible $IES = 2$, together with a lower-than-unitary elasticity of substitution between capital and labor inputs in firms' production, our calibrated one-sector endogenous growth model is able to generate qualitatively as well as quantitatively realistic long-run net-income-inequality effects of progressive taxation *vis-à-vis* Duncan and Sabirianova Peter's (2016) empirical findings under useless or useful government spending.

6 Conclusion

A recent empirical study by Duncan and Sabirianova Peter (2016) finds that an increase in the tax progressivity will generate a decrease in net-income inequality, with the calculated elasticities of after-tax Gini with respect to the average rate progression ranging over the interval [0.1206, 0.411]. Motivated by this stylized fact, our paper examines the long-run distributional impacts of more progressive taxation on agents' disposable income, not only theoretically but also quantitatively, in a one-sector endogenous growth model with heterogeneous households, a constant elasticity-of-substitution production technology and wasteful government spending. In a simplified two-type version of the macroeconomy, we analytically show that higher fiscal progression leads to less unequal distributions of gross/net income in the long run and a lower output growth rate, provided the general-equilibrium elasticity of aggregate labor supply exceeds a specific negative threshold. In a calibrated model economy, our baseline setting correctly yields that a higher degree of tax progressivity will reduce the steady-state dispersion of post-tax income, but the resulting calculated elasticity is too low to be empirically realistic. In light of this finding, we conduct sensitivity analyses for our benchmark model as well as an otherwise identical endogenously growing macroeconomy with useful government purchases of goods and services. These numerical simulations demonstrate that our calibrated one-sector endogenous growth model under useless or useful public expenditures, along with (i) agents' intertemporal elasticity of consumption substitution taking on the highest possible value that is regarded as empirically plausible and (ii) a lower-than-unitary elasticity of substitution between capital and labor inputs in production, is able to generate the long-run net-income-inequality effects of changing fiscal progression that are qualitatively as well as quantitatively consistent with recent panel estimation results.

Our analysis can be extended in several directions. In particular, it would be worthwhile to explore alternative mechanisms for generating sustained economic growth (*e.g.* human capital accumulation) and/or an economy with national debt or multiple production sectors. Moreover, it would be valuable to investigate the distributional consequences of linearly progressive income taxation *à la* Dromel and Pintus (2007). These possible extensions will allow us to examine the robustness of this paper's theoretical and numerical results, as well as further enhance our understanding of the interrelations between progressive taxation and income inequality within an endogenously growing macroeconomy.

7 Appendix A

Proof of Proposition 1. Since the BGP growth rate of income (35) is identical across all households, the equality of $\tilde{\Phi}_{\tilde{Y}_i}$ and $\tilde{\Phi}_{\tilde{Y}_j}$ yields

$$\beta_i - \beta_j = \alpha A[\alpha + (1-\alpha)\tilde{H}^{\varepsilon}]^{\frac{1-\varepsilon}{\varepsilon}} \underbrace{\left[\eta(1+\phi)(\tilde{y}_j^{\phi} - \tilde{y}_i^{\phi})\right]}_{= \tilde{\tau}_j^m - \tilde{\tau}_i^m}, \tag{A.1}$$

which implies that the more patient agent with a lower time preference rate $\beta_j < \beta_i$ will have a higher steady-state share of gross income $\tilde{y}_j > \tilde{y}_i$ and face a higher marginal tax rate $\tilde{\tau}_j^m > \tilde{\tau}_i^m$. Since each household's after-tax income is monotonically increasing in its before-tax counterpart, we then find that $(1-\tilde{\tau}_j)\tilde{Y}_j > (1-\tilde{\tau}_i)\tilde{Y}_i$ and thus $\tilde{y}_j^a > \tilde{y}_i^a$.

Next, equation (30) can be re-written as the following continuous function:

$$F(\tilde{y}_i, \tilde{H}_i, \tilde{H}) \equiv \tilde{y}_i - (1-\alpha)\frac{\tilde{H}_i}{\tilde{H}} - \alpha \left\{ \frac{(1-\tilde{\tau}_i)\tilde{y}_i - \frac{(1-\alpha)(1-\tilde{\tau}_i^m)(1-\tilde{H}_i)}{\theta[\alpha+(1-\alpha)\tilde{H}^{\varepsilon}]\tilde{H}^{1-\varepsilon}}}{(1-\bar{\tau}) - \frac{(1-\alpha)(1-\bar{\tau}^m)(1-\tilde{H})}{\theta[\alpha+(1-\alpha)\tilde{H}^{\varepsilon}]\tilde{H}^{1-\varepsilon}}} \right\} = 0. \tag{A.2}$$

Using the intermediate value theorem, we further obtain that

$$F(\tilde{y}_i, \tilde{H}_i, \tilde{H}) = 0 = F(\tilde{y}_j,\ \tilde{H}_j,\ \tilde{H}) = \left.\frac{\partial F}{\partial \tilde{y}_i}\right|_{\tilde{y}_s, \tilde{H}_s} (\tilde{y}_i - \tilde{y}_j) + \left.\frac{\partial F}{\partial \tilde{H}_i}\right|_{\tilde{y}_s, \tilde{H}_s} (\tilde{H}_i - \tilde{H}_j), \tag{A.3}$$

where $\tilde{y}_s \in (\tilde{y}_i,\ \tilde{y}_j)$ and $\tilde{H}_s \in (\tilde{H}_i,\ \tilde{H}_j)$. As a result,

$$\tilde{y}_i - \tilde{y}_j = -\left\{ \frac{\left.\frac{\partial F}{\partial \tilde{H}_i}\right|_{\tilde{y}_s, \tilde{H}_s}}{\left.\frac{\partial F}{\partial \tilde{y}_i}\right|_{\tilde{y}_s, \tilde{H}_s}} \right\} (\tilde{H}_i - \tilde{H}_j). \tag{A.4}$$

Using equation (A.2), it is straightforward to show that

$$\frac{\partial F}{\partial \tilde{H}_i} = -\left\{ \frac{(1-\alpha)}{\tilde{H}} + \frac{\frac{\alpha(1-\alpha)(1-\tilde{\tau}_i^m)}{\theta[\alpha+(1-\alpha)\tilde{H}^{\varepsilon}]\tilde{H}^{1-\varepsilon}}}{(1-\bar{\tau}) - \frac{(1-\alpha)(1-\bar{\tau}^m)(1-\tilde{H})}{\theta[\alpha+(1-\alpha)\tilde{H}^{\varepsilon}]\tilde{H}^{1-\varepsilon}}} \right\}, \tag{A.5}$$

where $(1-\bar{\tau}) - \frac{(1-\alpha)(1-\bar{\tau}^m)(1-\tilde{H})}{\theta[\alpha+(1-\alpha)\tilde{H}^{\varepsilon}]\tilde{H}^{1-\varepsilon}} = \frac{\tilde{\Phi}+\delta}{A[\alpha+(1-\alpha)\tilde{H}^{\varepsilon}]^{\frac{1}{\varepsilon}}} > 0$ from the steady-state version of equation (32), which in turn leads to $\frac{\partial F}{\partial \tilde{H}_i} < 0$; and that

$$\frac{\partial F}{\partial \tilde{y}_i} = 1-\alpha \left\{ \frac{1-\tilde{\tau}_i^m + \frac{(1-\alpha)\phi\tilde{\tau}_i^m(1-\tilde{H}_i)}{\theta[\alpha+(1-\alpha)\tilde{H}^\varepsilon]\tilde{H}^{1-\varepsilon}}}{1-\bar{\tau} - \frac{(1-\alpha)(1-\bar{\tau}^m)(1-\tilde{H})}{\theta[\alpha+(1-\alpha)\tilde{H}^\varepsilon]\tilde{H}^{1-\varepsilon}}} \right\} = \frac{\tilde{\Phi}+\delta-(1-\tilde{\tau}_i^m)\tilde{r} - \alpha\left\{\frac{(1-\alpha)\phi\tilde{\tau}_i^m(1-\tilde{H}_i)}{\theta A[\alpha+(1-\alpha)\tilde{H}^\varepsilon]^{\frac{\varepsilon-1}{\varepsilon}}\tilde{H}^{1-\varepsilon}}\right\}}{\tilde{\Phi}+\delta}, \tag{A.6}$$

where $\tilde{\Phi}+\delta-(1-\tilde{\tau}_i^m)\tilde{r} = -\beta_i < 0$ per the common BGP growth rate given by (35). It follows that $\frac{\partial F}{\partial \tilde{y}_i} < 0$ as well. We then substitute $\frac{\partial F}{\partial \tilde{H}_i} < 0$ and $\frac{\partial F}{\partial \tilde{y}_i} < 0$ into equation (A.4) to find that the more patient agent j with a higher gross income share $\tilde{y}_j > \tilde{y}_i$ will supply a lower amount of labor hours $\tilde{H}_j < \tilde{H}_i$.

On the other hand, equation (30) yields that

$$\alpha(\tilde{k}_i - \tilde{k}_j) = (\tilde{y}_i - \tilde{y}_j) - (1-\alpha)\left(\frac{\tilde{H}_i - \tilde{H}_j}{\tilde{H}}\right). \tag{A.7}$$

Since the differences in gross income share $\tilde{y}_i - \tilde{y}_j$ and labor supply $\tilde{H}_i - \tilde{H}_j$ are of opposite signs, the above condition implies that the more patient households j with $\tilde{y}_j > \tilde{y}_i$ and $\tilde{H}_j < \tilde{H}_i$ will own relatively more capital/wealth than agent i, hence $\tilde{k}_j > \tilde{k}_i$. Our preceding derivations thus show that when $\beta_i > \beta_j$, equations (A.1)-(A.7) altogether lead to $\tilde{y}_j > \tilde{y}_i$, $\tilde{y}_j^a > \tilde{y}_i^a$, $\tilde{H}_j < \tilde{H}_i$ and $\tilde{k}_j > \tilde{k}_i$. It is then straightforward to extend this $\{i, j\}$-pair result to an economy with $M \geq 3$ groups of individuals and $\beta_1 > \beta_2 > \cdots > \beta_M$.■

8 Appendix B

Proof of Proposition 2. Per the same procedure as in Appendix A, the equality of $\tilde{\Phi}_{\tilde{Y}_1}$ and $\tilde{\Phi}_{\tilde{Y}_2}$ leads to

$$\beta_1 - \beta_2 = \alpha A[\alpha + (1-\alpha)\tilde{H}^\varepsilon]^{\frac{1-\varepsilon}{\varepsilon}} \underbrace{\left[\eta(1+\phi)(\tilde{y}_2^\phi - \tilde{y}_1^\phi)\right]}_{=\tilde{\tau}_2^m - \tilde{\tau}_1^m}, \tag{B.1}$$

where $\beta_1 > \beta_2$ and $\tilde{y}_1 + \tilde{y}_2 = 2$, which in turn implies that $\tilde{y}_1 < 1 < \tilde{y}_2$.

After taking logarithm on both sides of equation (B.1), it is straightforward to derive that *ceteris paribus* the impact of a change in the tax-progressivity parameter ϕ on type-1 agents' relative gross income share is given by

$$\frac{\partial \tilde{y}_1}{\partial \phi} = \frac{1}{\phi(\tilde{y}_1^{\phi-1} + \tilde{y}_2^{\phi-1})} \left\{ \underbrace{(\tilde{y}_2^\phi - \tilde{y}_1^\phi)}_{\text{Positive}} \left[\frac{1}{1+\phi} + \frac{(1-\alpha)(1-\varepsilon)\tilde{H}^\varepsilon}{\phi[\alpha+(1-\alpha)\tilde{H}^\varepsilon]} \cdot E_\phi^H\right] + (\tilde{y}_2^\phi \ln \tilde{y}_2 - \tilde{y}_1^\phi \ln \tilde{y}_1) \right\}, \tag{B.2}$$

where E_ϕ^H denotes the elasticity of aggregate steady-state labor supply $\tilde{H}$ with respect to the progressive level of income taxation. Since $(\tilde{y}_2^\phi \log \tilde{y}_2 - \tilde{y}_1^\phi \log \tilde{y}_1)$ is strictly positive, $\frac{\partial \tilde{y}_1}{\partial \phi} > 0$ results if the sufficient condition $E_\phi^H > \underline{E} \equiv -\frac{[\alpha+(1-\alpha)\tilde{H}^\varepsilon]\phi}{(1-\alpha)(1-\varepsilon)(1+\phi)\tilde{H}^\varepsilon}$ holds. In addition, using $\tilde{y}_1 + \tilde{y}_2 = 2$ and $Gini = 1 + \frac{1}{2}(1 - 2\tilde{y}_1 - \tilde{y}_2)$ yields that $\frac{\partial \tilde{y}_2}{\partial \phi} = -\frac{\partial \tilde{y}_1}{\partial \phi} < 0$ and

$$\frac{\partial Gini}{\partial \phi} = -\frac{1}{2}\frac{\partial \tilde{y}_1}{\partial \phi} < 0, \tag{B.3}$$

indicating that before-tax income inequality will fall/rise in response to a higher/lower degree of fiscal progression.

According to $\tilde{y}_1^a = \frac{(1-\eta\tilde{y}_1^\phi)\tilde{y}_1}{1-\frac{\eta}{2}(\tilde{y}_1^{1+\phi}+\tilde{y}_2^{1+\phi})}$ and $\tilde{y}_2^a = \frac{(1-\eta\tilde{y}_2^\phi)\tilde{y}_2}{1-\frac{\eta}{2}(\tilde{y}_1^{1+\phi}+\tilde{y}_2^{1+\phi})}$, it can be shown that

$$\begin{aligned}\underbrace{(1-\bar{\tau})^2}_{\text{Positive}}\left[\frac{\partial \tilde{y}_2^a}{\partial \phi} - \frac{\partial \tilde{y}_1^a}{\partial \phi}\right] = \underbrace{\frac{\partial \tilde{y}_1}{\partial \phi}}_{\text{Positive}} & \left[(1-\bar{\tau})\underbrace{(\tilde{\tau}_1^m + \tilde{\tau}_2^m - 2)}_{\text{Negative}} + \frac{\eta}{2}(1+\phi)\underbrace{(\tilde{y}_1^\phi - \tilde{y}_2^\phi)}_{\text{Negative}}\underbrace{[(1-\tilde{\tau}_2)\tilde{y}_2 - (1-\tilde{\tau}_1)\tilde{y}_1]}_{\text{Positive}}\right] \\ & - \eta\tilde{y}_1\tilde{y}_2\underbrace{\left[(1-\tilde{\tau}_1)\tilde{y}_2^\phi \log \tilde{y}_2 - (1-\tilde{\tau}_2)\tilde{y}_1^\phi \log \tilde{y}_1\right]}_{\text{Positive}} < 0,\end{aligned} \tag{B.4}$$

where $0 < \tilde{\tau}_1, \tilde{\tau}_2, \tilde{\tau}_1^m, \tilde{\tau}_2^m, \bar{\tau} < 1$, $\log \tilde{y}_1 < 0 < \log \tilde{y}_2$ and the requisite condition $E_\phi^H > \underline{E}$ is postulated to be satisfied such that $\frac{\partial \tilde{y}_1}{\partial \phi} > 0$. As a result, the difference between patient/rich versus impatient/poor households' relative net-income shares will become smaller, *i.e.* $\frac{\partial \tilde{y}_2^a}{\partial \phi} - \frac{\partial \tilde{y}_1^a}{\partial \phi} < 0$, which in turn yields a lower after-tax Gini coefficient with

$$\begin{aligned}\underbrace{\left[4(1-\bar{\tau})^2\right]}_{\text{Positive}}\frac{\partial Gini^a}{\partial \phi} = -\underbrace{\frac{\partial \tilde{y}_1}{\partial \phi}}_{\text{Positive}} & \left\{2(1-\bar{\tau}) + \eta(1+\phi)\left[\eta\,(\tilde{y}_1\tilde{y}_2)^\phi\,(\tilde{y}_1 + \tilde{y}_2) + (\tilde{y}_2^\phi - \tilde{y}_1^\phi)[2(1-\tilde{y}_1) + (2-\tilde{y}_2)]\right]\right\} \\ & - \eta\tilde{y}_1\tilde{y}_2\underbrace{\left[(1-\tilde{\tau}_1)\tilde{y}_2^\phi \log \tilde{y}_2 - (1-\tilde{\tau}_2)\tilde{y}_1^\phi \log \tilde{y}_1\right]}_{\text{Positive}} < 0,\end{aligned} \tag{B.5}$$

where $Gini^a = 1 + \frac{1}{2}(1 - 2\tilde{y}_1^a - \tilde{y}_2^a)$ and thus $\frac{\partial Gini^a}{\partial \phi} < 0$.

Finally, we use the common BGP growth rate (35) for type-1 individuals to obtain that

$$
\underbrace{\left[\frac{1-\gamma}{\alpha A[\alpha+(1-\alpha)\tilde{H}^{\varepsilon}]^{\frac{1-\varepsilon}{\varepsilon}}}\right]}_{\text{Positive}}\frac{\partial\tilde{\Phi}}{\partial\phi} = -\left[\eta\tilde{y}_1^{\phi} + \eta(1+\phi)\,(\tilde{y}_1\tilde{y}_2)^{\phi-1}\underbrace{(\tilde{y}_1\log\tilde{y}_1 + \tilde{y}_2\log\tilde{y}_2)}_{(A)}\right]
$$

$$
-\,\eta(1+\phi)\tilde{y}_1^{\phi-1}(\tilde{y}_2^{\phi}-\tilde{y}_1^{\phi})\underbrace{\left[\frac{1}{1+\phi}+\frac{(1-\alpha)(1-\varepsilon)\tilde{H}^{\varepsilon}}{[\alpha+(1-\alpha)\tilde{H}^{\varepsilon}]\phi}\cdot E_{\phi}^{H}\right]}_{(B)}
$$

$$
+\,\frac{(1-\alpha)(1-\varepsilon)(1-\tilde{\tau}_1^{m})(\tilde{y}_1^{\phi-1}+\tilde{y}_2^{\phi-1})\tilde{H}^{\varepsilon}}{\phi[\alpha+(1-\alpha)\tilde{H}^{\varepsilon}]}\cdot\underbrace{E_{\phi}^{H}}_{\text{Negative}}, \qquad \text{(B.6)}
$$

where $(A) \geqslant 0$ by the Jensen's inequality, $(B) > 0$ due to the sufficient condition $E_{\phi}^{H} > \underline{E}$ for $\frac{\partial\tilde{y}_1}{\partial\phi} > 0$ to hold, and $E_{\phi}^{H} < 0$ because of a stronger substitution (*c.f.* income) effect under our postulated homogenous utility function (6). It follows that the right-hand side of equation (B.6) is negative, and hence the economy's output growth rate will fall/rise when the tax schedule becomes more/less progressive, *i.e.* $\frac{\partial\tilde{\Phi}}{\partial\phi} < 0$.■

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Table 1. Benchmark Model with Useless Government Spending

under $\text{IES} = \frac{1}{3}$ $(\gamma = -2)$ and $\text{ELS} = 0.87$ $(\varepsilon = -0.1494)$

	$ARP = 0.0362$ $(\phi = 0.3744)$	$ARP' = 0.0462$ $(\phi' = 0.381)$		$ARP = 0.0362$ $(\phi = 0.3744)$	$ARP' = 0.0462$ $(\phi' = 0.381)$
$\frac{\tilde{k}_1}{5}$	-0.3067	-0.3016	$\frac{\tilde{y}_1^a}{5}$	0.0805	0.0823
$\frac{\tilde{k}_2}{5}$	-0.0812	-0.0762	$\frac{\tilde{y}_2^a}{5}$	0.1329	0.1344
$\frac{\tilde{k}_3}{5}$	0.1061	0.1097	$\frac{\tilde{y}_3^a}{5}$	0.1745	0.1754
$\frac{\tilde{k}_4}{5}$	0.3554	0.3559	$\frac{\tilde{y}_4^a}{5}$	0.227541	0.227528
$\frac{\tilde{k}_5}{5}$	0.9264	0.9122	$\frac{\tilde{y}_5^a}{5}$	0.3846	0.3804
$\tilde{H}$	0.3	0.2992	$\tilde{\Phi}$	0.02	0.0198
$\frac{\tilde{H}_1}{5}$	0.1138	0.1131	$Gini^a$	0.2812	0.2758
$\frac{\tilde{H}_2}{5}$	0.0867	0.086	$\left\lvert \frac{\Delta Gini^a / Gini^a}{\Delta ARP / ARP} \right\rvert$	0.0693	
$\frac{\tilde{H}_3}{5}$	0.0644	0.0639			
$\frac{\tilde{H}_4}{5}$	0.0351	0.035			
$\frac{\tilde{H}_5}{5}$	$2.4481\text{x}10^{-5}$	$1.1868\text{x}10^{-3}$			

Table 2. Benchmark Model with Useless Government Spending: Sensitivity Analysis on IES and ELS

	$\text{IES} = \frac{1}{3}\left(\gamma = -2\right)$					
	ELS = 0.87 ($\varepsilon = -0.1419$)		ELS = 0.45 ($\varepsilon = -1.222$)		ELS = 0.3 ($\varepsilon = -2.333$)	
ARP	0.0362	0.0462	0.0362	0.0462	0.0362	0.0462
$Gini^a$	0.2812	0.27579	0.2812	0.275839	0.2812	0.27587
$\left\lvert \frac{\Delta Gini^a / Gini^a}{\Delta ARP / ARP} \right\rvert$	0.0693		0.06867		0.06824	

	$\text{IES} = 1\left(\gamma = 0\right)$					
	ELS = 0.87 ($\varepsilon = -0.1419$)		ELS = 0.45 ($\varepsilon = -1.222$)		ELS = 0.3 ($\varepsilon = -2.333$)	
ARP	0.0362	0.0462	0.0362	0.0462	0.0362	0.0462
$Gini^a$	0.2812	0.27511	0.2812	0.27495	0.2812	0.27481
$\left\lvert \frac{\Delta Gini^a / Gini^a}{\Delta ARP / ARP} \right\rvert$	0.07801		0.08016		0.08192	

	$\text{IES} = 2\left(\gamma = 0.5\right)$					
	ELS = 0.87 ($\varepsilon = -0.1419$)		ELS = 0.45 ($\varepsilon = -1.222$)		ELS = 0.3 ($\varepsilon = -2.333$)	
ARP	0.0362	0.0462	0.0362	0.0462	0.0362	0.0462
$Gini^a$	0.2812	0.27301	0.2812	0.27027	0.2812	0.26456
$\left\lvert \frac{\Delta Gini^a / Gini^a}{\Delta ARP / ARP} \right\rvert$	0.1051		0.14042		0.21395	

Table 3. Productive Government Spending with $\chi = 0.6$ and $Gini^a = 0.2812$ under $ARP = 0.0362$

	$\text{IES} = \frac{1}{3}\left(\gamma = -2\right)$		$\text{IES} = 1\left(\gamma = 0\right)$		$\text{IES} = 2\left(\gamma = 0.5\right)$	
$ARP = 0.0462$	$Gini^a$	$\left\lvert\frac{\Delta Gini^a / Gini^a}{\Delta ARP / ARP}\right\rvert$	$Gini^a$	$\left\lvert\frac{\Delta Gini^a / Gini^a}{\Delta ARP / ARP}\right\rvert$	$Gini^a$	$\left\lvert\frac{\Delta Gini^a / Gini^a}{\Delta ARP / ARP}\right\rvert$
ELS = 0.87	0.27577	0.06954	0.27489	0.08084	0.27125	0.12777
ELS = 0.45	0.275844	0.06861	0.27468	0.08357	0.26342	0.22858
ELS = 0.3	0.27589	0.06804	0.27453	0.08558	0.22927	0.66827

Table 4. Utility-Generating Government Spending with $\mu = 0.3$ and $Gini^a = 0.2812$ under $ARP = 0.0362$

	$\text{IES} = \frac{1}{3}\left(\gamma = -2\right)$		$\text{IES} = 1\left(\gamma = 0\right)$		$\text{IES} = 2\left(\gamma = 0.5\right)$	
$ARP = 0.0462$	$Gini^a$	$\left\lvert\frac{\Delta Gini^a / Gini^a}{\Delta ARP / ARP}\right\rvert$	$Gini^a$	$\left\lvert\frac{\Delta Gini^a / Gini^a}{\Delta ARP / ARP}\right\rvert$	$Gini^a$	$\left\lvert\frac{\Delta Gini^a / Gini^a}{\Delta ARP / ARP}\right\rvert$
ELS = 0.87	0.27583	0.06875	0.27511	0.07801	0.2663	0.1915
ELS = 0.45	0.27589	0.068	0.27495	0.08016	0.25824	0.32945
ELS = 0.3	0.27593	0.06752	0.27481	0.08192	0.25182	0.37796